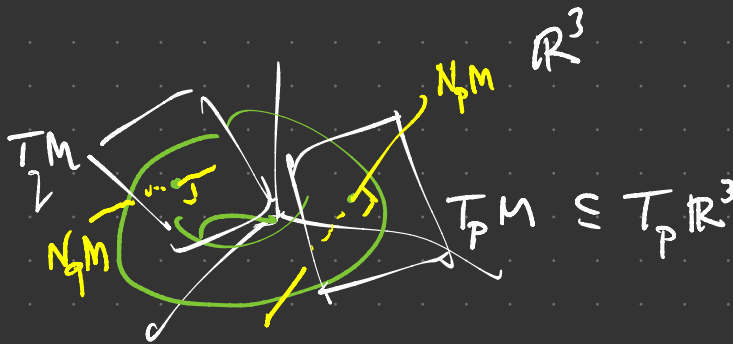


Review

Normal bundles / tubular nbhds

$M \subseteq \mathbb{R}^n$ embedded submfld



$$N_p M := (T_p M)^\perp$$

$$T_p \mathbb{R}^n \cong T_p M \oplus N_p M$$

$$\dots \left\{ \begin{array}{l} C_0(U \cup V) \\ C_0(U) \oplus C_0(V) \\ C_0(X) \end{array} \right.$$

$$0 \rightarrow C_0 \rightarrow D_0 \rightarrow E_0 \rightarrow 0$$

$\Rightarrow \exists$ natural LES

$$\dots \rightarrow H_n C_0 \rightarrow H_n D_0 \rightarrow H_n E_0 \rightarrow \dots$$

$$\rightarrow H_{n-1} C_0 \rightarrow H_{n-1} D_0 \rightarrow H_{n-1} E_0 \rightarrow \dots$$

C_0 de expo:

$$\dots \rightarrow C_{n+1} \xrightarrow{d} C_n \xrightarrow{d} C_{n-1} \xrightarrow{d} \dots$$

$$d \circ d = 0$$

$$H_n C_0 := \ker(d: C_n \rightarrow C_{n-1})$$

im($d: C_{n+1} \rightarrow C_n$)

$$\begin{array}{ccccccc} \dots & & & & & & \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \rightarrow & C_{n+1} & \rightarrow & D_{n+1} & \rightarrow & E_{n+1} \rightarrow 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ 0 & \rightarrow & C_n & \rightarrow & D_n & \rightarrow & E_n \rightarrow 0 \\ & & \downarrow & & \downarrow & & \downarrow \\ \vdots & & \vdots & & \vdots & & \vdots \end{array}$$

$[e] \in H_{n+1} \bar{E}$. WTD $\partial[e] \in H_n C$. Take $e \in \ker(d: \bar{E}_{n+1} \rightarrow \bar{E}_n)$
 rep'ing $[e]$
 $= \ker(d: \bar{E}_{n+1} \rightarrow \bar{E}_n) / \text{im}(d: \bar{E}_{n+2} \rightarrow \bar{E}_{n+1})$

$$\begin{array}{ccccccc}
 0 & \longrightarrow & C_{n+1} & \longrightarrow & D_{n+1} & \longrightarrow & \bar{E}_{n+1} & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow d & & \downarrow d & & \\
 0 & \longrightarrow & C_n & \longrightarrow & D_n & \longrightarrow & \bar{E}_n & \longrightarrow & 0 \\
 & & & & \downarrow c & & \downarrow d(\delta) & & \\
 & & & & & & 0 & &
 \end{array}$$

A commutative diagram with two rows of exact sequences. The top row is $0 \rightarrow C_{n+1} \rightarrow D_{n+1} \rightarrow \bar{E}_{n+1} \rightarrow 0$. The bottom row is $0 \rightarrow C_n \rightarrow D_n \rightarrow \bar{E}_n \rightarrow 0$. Vertical arrows connect $C_{n+1} \rightarrow C_n$, $D_{n+1} \rightarrow D_n$ (labeled d), and $\bar{E}_{n+1} \rightarrow \bar{E}_n$ (labeled d). A yellow arrow labeled δ points from D_{n+1} to \bar{E}_{n+1} . A yellow arrow labeled c points from C_n to D_n . A yellow arrow labeled $d(\delta)$ points from D_n to \bar{E}_n . A yellow arrow labeled e points from \bar{E}_{n+1} to \bar{E}_n .

Define $\partial[e] = [c]$

$$X \rightsquigarrow C_0 X \rightsquigarrow H_n X := H_n(C_0 X)$$

singular chain cpx
of X

$$C_n X = \mathbb{Z} \{ \sigma : \Delta^n \rightarrow X \mid \sigma \text{ cts} \}$$

$$d \downarrow \\ C_{n-1} X$$

$\ker d = \text{"cycles"}$
 $\text{im } d = \text{"boundaries"}$



$$X \xrightarrow{f} Y \quad \rightsquigarrow \quad C_*(X) \xrightarrow{f_\#} C_*(Y)$$

$$H_n X \xrightarrow{f_*} H_n Y$$

$$\begin{array}{ccc} \sigma: \Delta^n & \rightarrow & X \\ & \searrow f_\# \sigma & \downarrow f \\ & & Y \end{array}$$

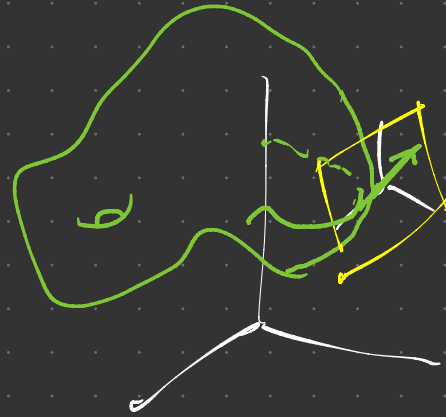
Hurewicz

$$\pi_1(X, x) \longrightarrow H_1(X) = \pi_1 X^{ab}$$

$\downarrow \exists!$
 A

$$M \subseteq \mathbb{R}^n$$

$$T_p M \subseteq T_p \mathbb{R}^n = \mathbb{R}^n$$



\mathbb{R}^n
" $S \subseteq M$ emb submanifold

$$T_p S \subseteq T_p M$$

$$T_p S = \left\{ v \in T_p M \mid v f = 0 \forall f \in C^\infty(M) \right. \\ \left. \text{s.t. } f|_S = 0 \right\}$$

$\underbrace{\mathbb{R}^n}$
 $\text{span} \left\{ \frac{\partial}{\partial x^1}, \dots, \frac{\partial}{\partial x^n} \right\}$

$$T_p S \subseteq T_p \mathbb{R}^n = \left\{ \gamma'(0) \mid \gamma: J \rightarrow \mathbb{R}^n, \gamma(0) = p \right\}$$

$$\text{"} \\ \left\{ \gamma'(0) \mid \gamma: J \rightarrow S, \gamma(0) = p \right\}$$