MATH 545: MANIFOLDS
HOMEWORK DUE FRIDAY WEEK 10

Problems taken from *Introduction to Smooth Manifolds* are marked ISM x–y. Please review the syllabus for expectations and policies regarding homework.

Problem 1 (ISM 6–3). Let M be a smooth manifold, let $B \subseteq M$ be a closed subset, and let $\delta \colon M \to \mathbb{R}$ be a positive continuous function. Show that there is a smooth function $\tilde{\delta} \colon M \to \mathbb{R}$ that is zero on B, positive on $M \smallsetminus B$, and satisfies $\tilde{\delta}(x) < \delta(x)$ everywhere. [*Hint*: Consider $f/(f+1)$, where f is a smooth nonnegative function that vanishes exactly on B, and use ISM Corollary 6.22.]

Problem 2 (ISM 6–4). Let M be a smooth manifold, let B be a closed subset of M, and let $\delta \colon M \to \mathbb{R}$ be a positive continuous function.

(a) Given any continuous function $f \colon M \to \mathbb{R}^k$, show that there is a continuous function $\tilde{f} \colon M \to \mathbb{R}^k$ that is smooth on $M \smallsetminus B$, agrees with f on B, and is δ-close to f. (*Hint*: Use the previous problem.)
(b) Given a smooth manifold N and a continuous map $F \colon M \to N$, show that F is homotopic relative B to a map that is smooth on $M \smallsetminus B$.

Problem 3 (ISM 6–8). Prove that every proper continuous map between smooth manifolds is homotopic to a proper smooth map. [*Hint*: Show that the map $\tilde{F}$ constructed in the proof of ISM Theorem 6.26 is proper if F is.]

Problem 4 (ISM 6–10). Suppose $F \colon N \to M$ is a smooth map that is transverse to an embedded submanifold $X \subseteq M$, and let $W = F^{-1}X$. For each $p \in W$, show that $T_pW = (dF_p)^{-1}(T_{F(p)}X)$. Conclude that if two embedded submanifolds $X, X' \subseteq M$ intersect transversely, then $T_p(X \cap X') = T_pX \cap T_pX'$ for every $p \in X \cap X'$.