MATH 545: MANIFOLDS
HOMEWORK DUE FRIDAY WEEK 8

Problems taken from *Introduction to Smooth Manifolds* are marked ISM x–y. Please review the syllabus for expectations and policies regarding homework.

Problem 1 (ISM 4–8). This problem shows that the converse of ISM Theorem 4.29 is false. Let $\pi\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $\pi(x, y) = xy$. Show that π is surjective and smooth, and for each smooth manifold P, a map $F\colon \mathbb{R} \to P$ is smooth if and only if $F \circ \pi$ is smooth; but π is not a smooth submersion.

Problem 2 (ISM 4–12). Using the covering map $\varepsilon^2\colon \mathbb{R}^2 \to \mathbb{T}^2$, show that the immersion $X\colon \mathbb{R}^2 \to \mathbb{R}^3$ defined in ISM Example 4.2(d) descends to a smooth embedding of $\mathbb{T}^2$ into $\mathbb{R}^3$. Specifically, show that X passes to the quotient to define a smooth map $\tilde{X}\colon \mathbb{T}^2 \to \mathbb{R}^3$, and then show that $\tilde{X}$ is a smooth embedding whose image is the given surface of revolution.

Problem 3 (ISM 4–13). Define a map $F\colon S^2 \to \mathbb{R}^4$ by $F(x, y, z) = (x^2 - y^2, xy, xz, yz)$. Using the standard smooth covering map $q\colon S^2 \to \mathbb{RP}^2$, show that F descends to a smooth embedding of $\mathbb{RP}^2$ into $\mathbb{R}^4$.