MATH 545: MANIFOLDS
HOMEWORK DUE FRIDAY WEEK 2

Problems taken from *Introduction to Topological Manifolds* are marked ITM x–y. Please review the syllabus for expectations and policies regarding homework.

Problem 1. Let $G = \mathfrak{S}_3$ be the symmetric group on three letters and note that $S = \{(1\ 2), (1\ 2\ 3)\}$ generates G.

(a) Draw the Cayley graph $\Gamma_G = \Gamma_{G,S}$ for G relative to S.
(b) Use your Cayley graph to argue that

$$G \cong \langle a, b \mid a^3 = b^2 = (ab)^2 = e \rangle.$$

(c) Describe the Cayley complex $\tilde{X}_G$ of G relative to S. (It might be hard to draw a picture, but you could try, and you should write precisely regarding how many 2-cells are attached and in what fashion.)
(d) Describe the left action of G on $\tilde{X}_G$. (It is enough to specify how the generators act at the level of 2-cells.)
(e) Verify in this case that $\tilde{X}_G/G \cong X_G$.

Problem 2 (ITM 11–1). Suppose $q\colon E \to X$ is a covering map.

(a) Show that if X is Hausdorff, then E is too.
(b) Show that if X is an n-manfiold, then E is too.
(c) Show that if E is an n manifold and X is Hausdorff, then X is an n-manifold.

Problem 3 (ITM 11–4). Construct a two-sheeted covering of the Klein bottle by the torus.

Problem 4. Suppose groups G_1, G_2 act continuously on spaces X_1, X_2, respectively, such that the quotient maps $q_i\colon X_i \to X_i/G_i$ are covering maps for $i = 1, 2$.

(a) Show that the action of $G_1 \times G_2$ on $X_1 \times X_2$ defined by $(g_1, g_2) \cdot (x_1, x_2) = (g_1 x_1, g_2 x_2)$ makes the quotient map $X_1 \times X_2 \to (X_1 \times X_2)/(G_1 \times G_2)$ a covering map.
(b) Prove that $(X_1 \times X_2)/(G_1 \times G_2) \cong X_1/G_1 \times X_2/G_2$.

Problem 5. Let $p_3\colon S^1 \to S^1$ denote the third power map $z \mapsto z^3$.

(a) Explicitly describe the monodromy action of $\pi_1(S^1, 1)$ on $p_3^{-1}\{1\}$.
(b) What is the isotropy group of each element of $p_3^{-1}\{1\}$?
(c) What is the induced subgroup $(p_3)_*\pi_1(S^1, 1) \leq \pi_1(S^1, 1)$?