MATH 544: TOPOLOGY
HOMEWORK DUE FRIDAY WEEK 11

Problems taken from *Introduction to Topological Manifolds* are marked ITM x–y. Please review the syllabus for expectations and policies regarding homework.

Problem 1 (ITM 10–1). Use the Seifert–van Kampen theorem to give another proof that S^n is simply connected when $n \geq 2$.

Problem 2 (ITM 10–2). Let

$$X := S^2 \cup \{(0,0,z) \mid -1 \leq z \leq 1\} \subseteq \mathbb{R}^3$$

and let $N = (0,0,1) \in X$. Compute $\pi_1(X, N)$, giving explicit generators.

Problem 3 (ITM 10–7). Suppose M and N are connected n-manifolds with $n \geq 3$. Prove that the fundamental group of $M \# N$ is isomorphic to $\pi_1(M) * \pi_1(N)$. [*Hint*: Prove ITM 4–19 and 10–6 and then use these results.]

Problem 4 (ITM 10–11). For each of the following spaces, give a presentation of the fundamental group together with a specific loop representing each generator.

(a) A closed disk with two interior points removed.
(b) The projective plane with two points removed.
(c) A connected sum of n tori with one point removed.
(d) A connected sum of n tori with two points removed.

Problem 5 (ITM 10–13). Let n be an integer greater than 2. Construct a polygonal presentation whose geometric realization has a fundamental group that is cyclic of order n.