

MATH 544: TOPOLOGY
HOMEWORK DUE FRIDAY WEEK 8

Problems taken from *Introduction to Topological Manifolds* are marked ITM x - y . Please review the syllabus for expectations and policies regarding homework.

Problem 1 (ITM 7–8). Prove that a retract of a Hausdorff space is a closed subset.

Problem 2 (ITM 7–14). Let M be a compact connected surface. Show that there is a point $p \in M$ such that $M \setminus \{p\}$ is homotopy equivalent to a bouquet of circles. (By convention, a bouquet of zero circles is a single point.)

Problem 3 (ITM 7–17). Show that the wedge sum of two pointed spaces is their coproduct in the category Top_* .

Problem 4 (ITM 7–18). Show that the direct sum of two Abelian groups is both their product and coproduct in the category Ab . Show that the infinite direct sum of Abelian groups¹ is the coproduct in Ab , while their Cartesian product is the product in Ab .

Problem 5 (ITM 7–19). Show that direct sum does not yield the coproduct in Grp . (You can follow the strategy in the text or invent your own.)

¹Given a family of Abelian groups $\{G_\alpha \mid \alpha \in A\}$, their *direct sum* is the subset $\bigoplus_{\alpha \in A} G_\alpha \subseteq \prod_{\alpha \in A} G_\alpha$ consisting of those A -tuples $(g_\alpha)_{\alpha \in A}$ such that $g_\alpha = 0$ for all but finitely many α .