MATH 544: TOPOLOGY
HOMEWORK DUE FRIDAY WEEK 5

Problems taken from *Introduction to Topological Manifolds* are marked ITM x–y. Please review the syllabus for expectations and policies regarding homework. It is recommended that you work out ITM 5–3 as well, but you don't need to turn anything in for this problem.

Problem 1 (ITM 4–27). Suppose X and Y are locally compact Hausdorff spaces. Show that a continuous map $f\colon X \to Y$ is proper if and only if it extends to a continuous map $f^*\colon X^* \to Y^*$ taking $\infty \in X^*$ to $\infty \in Y^*$. (Here X^* is the one-point compactification of X, introduced in last week's homework.)

Problem 2 (ITM 5–1). Suppose D and D' are closed cells (not necessarily of the same dimension).

(a) Show that every continuous map $f\colon \partial D \to \partial D'$ extends to a continuous map $F\colon D \to D'$ with $F(D^\circ) \subseteq (D')^\circ$.
(b) Given points $p \in D^\circ$ and $p' \in (D')^\circ$, show that F from (a) can be chosen so that $F(p) = p'$.
(c) Show that if f is a homeomorphism, then F can also be chosen to be a homeomorphism.

Problem 3 (ITM 5–8). Prove Proposition 5.7: If X is any CW complex, the topology of X is coherent with the collection of subspaces $\{X_n \mid n \in \mathbb{N}\}$.

Problem 4 (ITM 5-11). Prove Proposition 5.16: A CW complex is locally compact if and only if it is locally finite.

Problem 5 (ITM 5–12). Let $\mathbb{RP}^n$ be n-dimensional real projective space. The usual inclusion $\mathbb{R}^{k+1} \subseteq \mathbb{R}^{n+1}$ for $k < n$ allows us to consider $\mathbb{RP}^k$ as a subspace of $\mathbb{RP}^n$. Show that $\mathbb{RP}^n$ has a CW decomposition with one cell in each dimension $0, \ldots, n$ such that the k-skeleton is $\mathbb{RP}^k$ for $0 < k < n$. [*Hint*: You might use the hint in the book, or do things a different way.]