MATH 411: TOPICS IN ADVANCED ANALYSIS
HOMEWORK DUE WEDNESDAY WEEK 11

Problem 1. Suppose A is an LCA group and $f \in L^1_{bc}(A)$. Recall that for $s \in A$, $L_s f \in L^1_{bc}(A)$ is the function given by

$$L_s f(x) = f(s^{-1}x).$$

Find a nice formula for $\widehat{L_s f}$ and prove that it is true.

Problem 2. Let $S = [a_1, b_1] \times \cdots \times [a_d, b_d] \subseteq \mathbb{R}^d$ and let 1_S denote its characteristic function. Compute $\widehat{1_S}$.

Problem 3. Suppose $M\colon \mathbb{R}^d \to \mathbb{R}^d$ is a linear transformation, and $f \in L^1(\mathbb{R}^d)$. Prove that $\widehat{f \circ M}$ is given by

$$\widehat{f \circ M}(\xi) = \frac{1}{|\det M|}\hat{f}(M^{-\top}\xi)$$

where $M^{-\top}$ denotes the inverse transpose of M.

Problem 4. Suppose $\Delta \subseteq \mathbb{R}^2$ is a (solid) triangle with vertices in $\mathbb{Z}^2$. Show that the following properties are equivalent:

(a) Δ contains no other points of $\mathbb{Z}^2$ (in its interior or boundary),
(b) the area of Δ is $1/2$,
(c) Δ is **unimodular**, *i.e.*, $v_3 - v_1$ and $v_2 - v_1$ form a basis for $\mathbb{Z}^2$.

Hint: You might want to "double" the triangle to form a parallelogram.