

**MATH 411: TOPICS IN ADVANCED ANALYSIS**  
**HOMEWORK DUE WEDNESDAY WEEK 6**

*Problem 1.* Prove that if  $(f_n: [a, b] \rightarrow \mathbb{C})_{n \in \mathbb{N}}$  is a sequence of Riemann integrable functions converging uniformly to  $f: [a, b] \rightarrow \mathbb{C}$ , then

$$\lim_{n \rightarrow \infty} \int_a^b f_n(x) dx = \int_a^b f(x) dx.$$

*Problem 2.* Prove or disprove:

$$\lim_{n \rightarrow \infty} \lim_{k \rightarrow \infty} \frac{n/k}{n/k + 1} = \lim_{k \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{n/k}{n/k + 1}.$$

*Problem 3.* Let  $a < b$  be real numbers. Compute the Fourier transform of the characteristic function  $\chi_{[a, b]}$  of the closed interval  $[a, b]$ .

*Problem 4.* Compute the convolution

$$e^{-x^2} * e^{-x^2}.$$