

MATH 201: LINEAR ALGEBRA
HOMEWORK DUE FRIDAY WEEK 2

Make sure to review the homework instructions in the syllabus before writing your solutions. In particular, show your work and write in complete sentences (but also aim for concise explanations).

Problem 1. For each of the following systems of linear equations, do the following:

- Find the associated augmented matrix M .
- Compute the reduced row echelon form E for M . **Show your work as in class, specifying your row operations.**
- From E determine whether there are solutions to the system. If there is a unique solution, state it. If there are infinitely many solutions, express the set of solutions in two ways: (i) parametrically, as in examples 2.4 and 2.5 in Chapter One, Section I.2, and (ii) in vector form as in Chapter One, Section I.3.

(a)

$$\begin{aligned}x - 2y + z &= 1 \\ -4x + 2y - z &= 0 \\ 3x + 3y - z &= 1.\end{aligned}$$

(b)

$$\begin{aligned}x + y + 3z &= 3 \\ -x + y + z &= -1 \\ 2x + 3y + 8z &= 4.\end{aligned}$$

(c)

$$\begin{aligned}x - 2y + 2z &= 5 \\ x - y &= -1 \\ -x + y + z &= 5.\end{aligned}$$

(d)

$$\begin{aligned}2x - 2y - 3z &= -2 \\ 3x - 3y - 2z + 5w &= 7 \\ x - y - 2z - w &= -3.\end{aligned}$$

(e)

$$\begin{aligned}x + y + 3z &= 4 \\ x + 2y + 4z &= 5.\end{aligned}$$

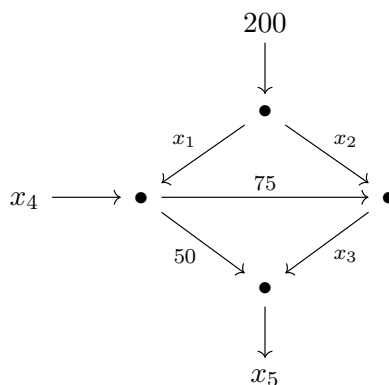
Problem 2. Some questions about conics.

- (a) Let $y = px^2 + qx + r$ be the equation of a general parabola. By solving a system of equations, find the constants p , q , and r so that the resulting parabola passes through the points $(-2, 15)$, $(1, 3)$, and $(2, 11)$.
- (b) A (real) plane conic is a set of points of the form

$$C = \{(x, y) \in \mathbb{R}^2 : ax^2 + bxy + cy^2 + dx + ey + f = 0\}$$

for some constants $a, b, c, d, e, f \in \mathbb{R}$, not all zero. For example, the unit circle centered at the origin is the conic specified by taking $a = c = 1$, $b = d = e = 0$, and $f = -1$ to get the defining equation, $x^2 + y^2 - 1 = 0$. Note that defining equation of a conic is only determined up to a scalar multiple: for instance, $2x^2 + 2y^2 - 2 = 0$, the conic with $a = c = 2$, $b = d = e = 0$, and $f = -2$, also determines the unit circle centered at the origin. The parabola specified by $y = px^2 + qx + r$ is a plane conic with parameters $a = p$, $b = c = 0$, $d = q$, $e = -1$, and $f = r$. Above, we saw an example where three points determined a parabola. How many points in the plane do you think must be specified to determine a conic, in general? Why? (Note: You probably don't have the tools yet to rigorously answer this question. Make a conjecture and explain your intuition.)

Problem 3. Consider the following network of one-way roads in which the labels on roads indicate the number of cars.



- Using the assumptions that (i) the flow into any node (intersection) equals the flow out of that intersection and (ii) the total flow into the network equals the total flow out, set up a system of equations modeling this road network.
- Solve the constraints of (a) by writing them as an augmented matrix and finding the reduced row echelon form. (You may give your answer in parametric or vector form.)
- Observing that flow rates must be nonnegative (these are one-way streets), determine bounds on the value of x_5 .

Problem 4. Review the syllabus and use this space to ask any questions you have about course policies (or write “I have reviewed the syllabus and have no questions” if that’s true).