CHAPTER **2**

Reading mathematics

Don't believe everything you read.
Anon

Obviously you can read and probably you have been taught reading skills for academic purposes as part of a study skills course. Unfortunately, mathematics has some special subtleties which often get missed in classes or books on how to study. For example, speed reading is recommended as a valuable tool for learning in many subjects. In mathematics, however, this is not a good method. Mathematics is rarely overwritten; there are few superfluous adjectives, every word and symbol is important and their omission would render the material incomprehensible or incorrect.

The hints and tips here, which include a systematic method for breaking down reading into digestible pieces, are practical suggestions, not a rigid list of instructions. The main points are the following:

- You should be flexible in your reading habits – read many different treatments of a subject.
- Reading should be a dynamic process – you should be an active, not passive, reader, working with a pen and paper at hand, checking the text and verifying what the author asserts is true.

The last point is where thinking mathematically diverges from thinking in many other subjects, such as history and sociology. You really do need to be following the details as you go along – check them. In history (assuming you don't have a time machine) you can't check that Caesar invaded Britain in 55 BC, you can only check what other people have claimed he did. In mathematics you really can, and should, verify the truth.

The following applies to reading lecture notes and web pages, not just to books, but to make a simpler exposition I shall refer only to books. Tips on specific situations, such as reading a definition, theorem[1] or proof are given in later chapters.

[1] A theorem is a mathematical statement that is true. Theorems will be discussed in greater detail in Part III.

Basic reading suggestions

Read with a purpose

The primary goal of reading is to learn, but we may be aiming to consolidate, clarify, or find an overview of some material.

Before reading decide what you want from the text. The goal may be as specific as learning a particular definition or how to solve a certain type of problem, such as integrating products. Whatever the reason, it is important that you do not start reading in the vague hope that everything will become clear.

How did you read the previous chapter of this book? What was your goal? Did you skim through it first to see if you already knew it? Did you want to read it in detail until you were confident that you understood everything? Answering these questions often gives an insight into what you really need to do when reading.

Choose a book at the right level

Some books are not well written and some may be unsuitable for your style of learning. In choosing a book bear in mind two connected points. Every book is written for an audience and a purpose. You may not be the audience, and the book's purpose, which might be to teach a novice or to be used as a reference for experts, may not match the purpose you require.

On the other hand do not reject advanced books totally since early chapters in a book often contain a useful summary of a subject.

Read with pen and paper at hand

Be active – read with pen and paper at hand.

The first reason for using pen and paper is that you should make notes from what you are reading – in particular, what it means, not what it says – and to record ideas as they occur to you. Don't take notes the first time you read through as you will probably copy too much without a lot of understanding.

The second reason is more important. You can explore theorems and formulas[2] by applying them to examples, draw diagrams such as graphs, solve – and even create your own – exercises. This is an important aspect of thinking like a mathematician. Physicists and chemists have laboratory experiments, mathematicians have these explorations as their experiments.

Reading with pen and paper at this stage excludes the use of fluorescent markers! The general tendency when using such pens is to mark everything, so wait until you need to summarize the text.

[2] Rather than use 'formulae', the correct latin plural of formula, I'll use a more natural English plural.

Don't read it like a novel

Do not read mathematics like a novel. You do not have to read from cover to cover or in the sequence presented. It is perfectly acceptable to dip in and out, take what is relevant to your situation, and to jump from page to page. This is perhaps surprising advice as mathematics is often thought of as a linear subject where ideas are built on top of one another. But, trust me, it isn't created in a linear way and it isn't learned in a linear way.

Add to this the fact that the tracks made by the pioneers of the subject have been covered and the presentation has been improved for public consumption, and you can probably see that you will need to skip backwards and forwards through a text.

Besides, it is unlikely that you will understand every detail in one sitting. You might have to read a passage a number of times before its true meaning becomes clear.

A systematic method

We now outline a five-point method for systematically tackling long pieces:

(i) Skim through and identify what is important.
(ii) Ask questions.
(iii) Read through carefully. You can do statements first and proofs later if you like.
(iv) Be active. This should include checking the text and doing the exercises.
(v) Reflect.

This is a simplistic system of reading which, though numbered, doesn't need to be slavishly followed in order. You may have to be flexible and jump from section to section depending on the situation.

Skim

First, look briefly through the text to get an overview. Study skills books often advise students to read the start and end of chapters to get the main conclusions. This does not always work in mathematics books as arguments are not usually summarized in this way, but it is worth trying.

Did you do this with Chapter 1? If I were to do this I would say that the main points are sets, numbers, operations on sets such union and intersection, functions and polynomials.

Identify what is important

In a more careful but not too detailed reading, identify the important points. Look for assumptions, definitions, theorems and examples that get used again and again, as these will be the key to illuminating the theory. If the same definition appears repeatedly in statements, it is important – so learn it!

From Chapter 1, for example, the concept of the empty set looks important, as does the necessity of discriminating between $\in$ and $\subseteq$, in particular their subtle difference.

Look for theorems or formulas that allow you to calculate because calculation is an effective way to get into a subject. Stop and reread that last sentence – I think it's one of the most useful pieces of advice given to me. Often when I am stuck trying to understand some theory attempting to calculate makes it clear. Noticing what allows you to calculate is thus very important.

In Chapter 1 the most obvious notion involving calculation was the cardinality of a set. However, there were no theorems involving it. Nonetheless, you should mark it as something that will be of use later because it involves the possibility of calculation. And in fact we look at calculation of cardinality in Chapter 5.

A more general example is the product rule and chain rule, etc. in calculus. These allow us to calculate the derivative of a function without using the definition of derivative (which is hard to work with).

Ask questions

At this stage it is helpful to pose some questions about the text, such as, Why does the theory hinge on this particular definition or theorem? What is the important result that the text is leading up to and how does it get us there? From your questions you can make a detailed list of what you want from the text.

In the last chapter the main point of the text was to lay the groundwork for material we will use later as examples.

Careful reading

It is now that the careful reading is undertaken. This should be systematic and combined with thinking, doing exercises and solving problems.

Reading is more than just reading the words, you must think about what they mean. In particular, ensure that you know the meaning of every word and symbol; if you don't know or have forgotten, then look back and find out.

For example, one needs to read carefully to ensure that the difference between being a set and being an element of a set is truly grasped.

Stop periodically to review

Do not try to read too much in one go. Stop periodically to review and think about the text. Keep thinking about the big picture, where are we going and how is a particular result getting us there?

Read statements first – proofs later

Many mathematical texts are written so that proofs can be ignored on an initial reading. This is not to say that proofs are unimportant; they are at the heart of mathematics, but usually – not always – can be read later. You *must* tackle the proof at some point.

There were no proofs in the previous chapter. Don't worry, we will produce many proofs later in the book.

Check the text

The necessity to check the text is why you need pen and paper at hand. There are two reasons. First, to fill in the gaps left by the writer. Often we meet phrases like 'By a straightforward calculation' or 'Details are left to the reader'. In that case, do that calculation or produce those details. This really allows you to get inside the theory.

For example, on page 7 in Chapter 1, I stated 'It is obvious that ... if $X \subseteq Y$, then $|X| \leq |Y|$.' Did you check to see that it really was 'obvious'? Did you try some examples? Similarly did you focus on the non-intuitive facts such as the fact that it is possible to have $\{x\} \in X$ and $\{x\} \subseteq X$ at the same time?

The second reason is to see how theorems, formulas, etc. apply. If the text says use Theorem 3.5 or equation Y, then check that Theorem 3.5 can be applied or check what happens to equation Y in this situation. Verify the formulas and so on. Be a sceptic – don't just take the author's word for it.

Do the exercises and problems

Most modern mathematics books have exercises and problems. It is hard to overplay the importance of doing these. Mathematics is an activity. Think of yourself as not studying mathematics, but *doing* mathematics.

Imagine yourself as having a mathematics muscle. It needs exercise to become developed. Passive reading is like watching someone else training with weights; it won't build your muscles – *you* have to do the exercises.

Furthermore, just because you have read something it does not mean you truly understand it. Answering the exercises and problems identifies your misconceptions and misunderstandings. Regularly I hear from students that they can understand a topic; it's just that they can't do the exercises, or can't apply the material. Basically, my rule is: if I can't do the exercises, then I don't understand the topic.

Reflect

In order to understand something fully we need to relate it to what we already know. Is it analogous to something else? For example, note how the $\subseteq$ notation made sense when it was compared with $\leq$ via cardinality. Can you think what intersection and union might be analogous to?

Another question to ask is 'What does this tell us or allow us to do that other work does not?' For example, the empty set allows us to count (something that was not explained but was alluded to in Chapter 1). Functions allow us to connect sets to sets. Cardinality allows us to talk about the relative sizes of sets. So when you meet a topic ask 'What does it allow me to do?'

What to do afterwards

Don't reread and reread – move on

It is unlikely that understanding will come from excessive rereading of a difficult passage. If you are rereading, then it is probably a sign that you are not active – so do some exercises, ask some questions and so on.

If that fails, it is time to look for an alternative approach, such as consulting another book. Ultimately, it is acceptable to give up and move on to the next part; you can always come back.

By moving on, you may encounter difficulty in understanding the subsequent material, but it might clarify the difficult part by revealing something important.

Also mathematics is a subject that requires time to be absorbed by the brain; ideas need to percolate and have time to grow and develop.

Reread

The assertion to reread may seem strange as the previous piece of advice was not to reread. The difference here is that one should come back much later and reread, for example, when you feel that you have learned the material. This often reveals many subtle points missed or gives a clearer overview of the subject.

Write a summary

The material may appear obvious once you have finished reading, but will that be true at a later date? It is a good time to make a summary – written in your own words.

Exercises

Exercises 2.1

(i) Look back at Chapter 1 and analyse how you attempted to read and understand it.

(ii) Find a journal or a science magazine that includes some mathematics in articles, for example *Scientific American*, *Nature*, or *New Scientist*.

Read an article. What is the aim of the article and who is the audience? How is the maths used? In one sentence what is the aim? Give three main points.

(iii) Find three textbooks of a similar level and within your mathematical ability. Briefly look through the books and decide which is most friendly and explain your reasons why.

(iv) Find three books tackling the same subject. Find a mathematical object in all three book, say, a set. Are the definitions different in the different books? Which is the best definition? Or rather, which is your favourite definition?

Are any diagrams used to illustrate the concept? What understanding do the diagrams give? How are the diagrams misleading?

Summary

- Read with a purpose.
- Read actively. Have pen and paper with you.
- You do not have to read in sequence but read systematically.
- Ask questions.
- Read the definitions, theorems and examples first. The proofs can come later.
- Check the text by applying formulas etc.
- Do exercises and problems.
- Move on if you are stuck.
- Write a summary.
- Reflect – What have you learned?