

**MATH 113: DISCRETE STRUCTURES
PRACTICE EXAM 2**

Question 1. Use induction to prove that for all $n \geq 1$,

$$\sum_{k=1}^n k(k+1) = \frac{1}{3}n(n+1)(n+2).$$

Question 2. Show that if 16 people are seated in a row of 20 chairs, then some group of 4 consecutive chairs must be occupied.

Question 3. Suppose that 6 people are standing in line. How many ways are there to rearrange the line so that nobody is standing in their original place?

Question 4. Let G be a graph that has exactly two vertices of odd degree, u and v . Prove there is a path in G from u to v .

BONUS

You should only attempt the bonus questions if you have completed the rest of the exam.

Question 5 (Bonus). Consider any five points in the plane that have integer coordinates. Prove that there are two points such that the midpoint of the line segment joining those two points also has integer coordinates.

Question 6 (Bonus). Let $n \geq 1$. Give a combinatorial proof of

$$\sum_{k=1}^n k(k+1) = \frac{1}{3}n(n+1)(n+2).$$