MATH 113: DISCRETE STRUCTURES
PRACTICE EXAM 1

Question 1. Don't forget to justify your answers. You want to send postcards to 12 friends. In the shop there are only 3 kinds of postcards. In how many ways can you send the postcards, if

(a) there is a large number of each kind of postcard, and you are sending exactly one postcard to each friend;
(b) there is a large number of each kind of postcard, and you are willing to send one or more postcards to each friend (but no one should get two identical cards);
(c) the shop has only 4 of each kind of postcard, and you want to send one card to each friend?

Question 2. Give a combinatorial proof of the following identity:

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}.$$

For partial credit, you can give an algebraic proof.

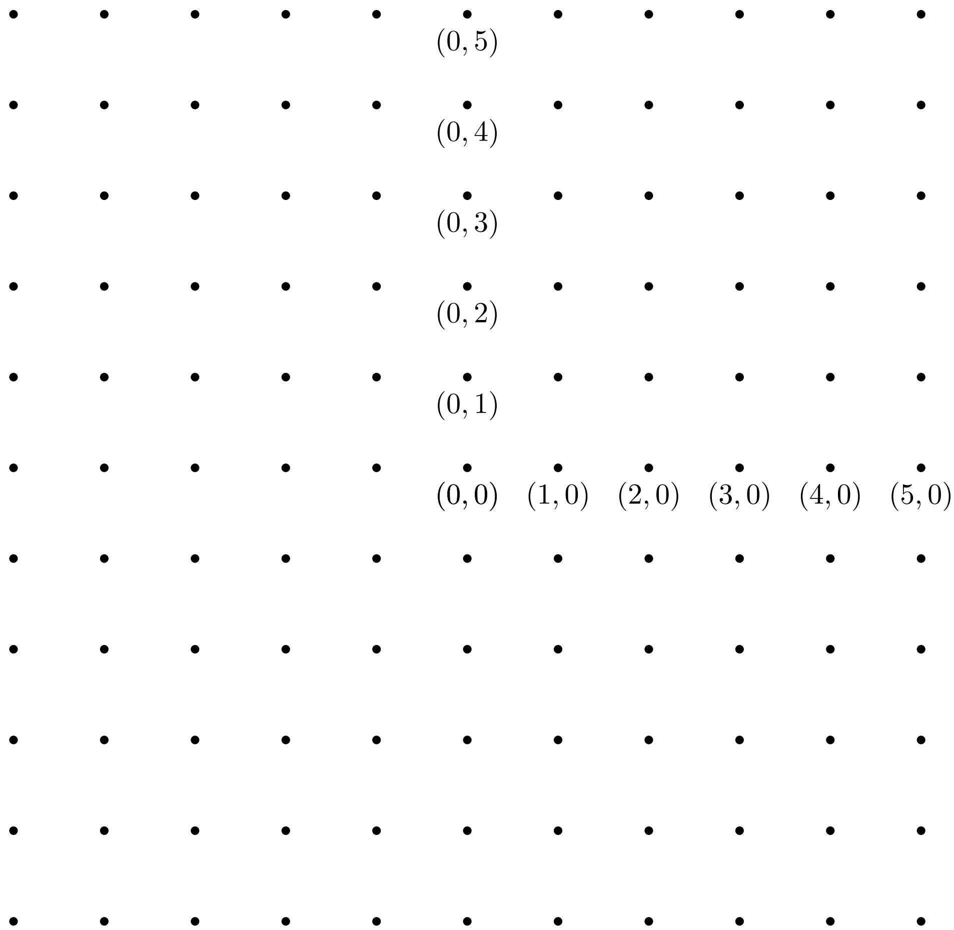
Question 3. Let $f\colon A \to B$ and $g\colon B \to C$ be functions. Prove that if f and g are surjective, then $g \circ f$ is also surjective.

Question 4. Let $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{N}$ be defined by

$$f(x, y) = |x| + |y|$$

where $|x|$ means 'the absolute value of x', so for example $f(-2, 3) = |-2| + |3| = 2 + 3 = 5$.

(a) Is f injective, surjective, or bijective? Prove your assertion for each property.
(b) We define a relation $\simeq_f$ on $\mathbb{Z} \times \mathbb{Z}$ by saying that $(x, y) \simeq_f (x', y')$ if $f(x, y) = f(x', y')$. State and prove **one** of the properties that $\simeq_f$ must have in order to be an equivalence relation.
(c) Consider the equivalence class $[(1, 3)]_{\simeq_f} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid (x, y) \simeq_f (1, 3)\}$. Circle all the elements of $[(1, 3)]_{\simeq_f}$ on the $\mathbb{Z} \times \mathbb{Z}$ grid below (you may instead write them as a set for partial credit). How many elements are in $[(1, 3)]_{\simeq_f}$?



(d) (BONUS:) Given $(x, y) \in \mathbb{Z} \times \mathbb{Z}$, how many elements are there in the equivalence class $[(x, y)]_{\simeq_f}$?