

**MATH 113: DISCRETE STRUCTURES
HOMEWORK 35**

Problem 1. Find, with proof, the remainder of $3^{1260126012601260129}$ upon division by 14.

Problem 2.

(a) For $n = 10, 11,$ and $12.$ List the fractions

$$\frac{1}{n}, \frac{2}{n}, \dots, \frac{n}{n}$$

after reducing each to lowest terms (canceling common factors in the numerator and denominator).

(b) Let n be a natural number and consider the quantity

$$\psi(n) = \sum_{d|n} \varphi(d)$$

which is the sum of the values $\varphi(d)$ where d ranges through all the positive divisors of $n.$ What is $\psi(n)$? (Experiment, formulate a conjecture, and prove it.) Your solution should consist of a precise statement of your conjecture and a proof. The proof does not need to be elaborate. It can just be a statement of the general relevant phenomenon you observe in part (a).

Problem 3. There are integers n such that -1 has a square root in $\mathbb{Z}/n\mathbb{Z}.$ To test this out, for each $n \in \{2, 3, \dots, 13\},$ find all solutions $x \in \{0, 1, \dots, n-1\}$ to the equation

$$x^2 \equiv -1 \pmod{n}.$$

You do not need to show your work. It may help to note that $-1 \equiv n-1 \pmod{n}.$