MATH 113: DISCRETE STRUCTURES
HOMEWORK 34

Problem 1.

(a) Find the smallest positive integer n such that $7^n \equiv 1 \pmod{100}$.

(b) Use your solution to part (a) to find the last two digits of 7^{2022}. (You can use a computer to check your answer, but show how the solution can be derived easily by hand using part (a).)

(c) (BONUS) What are the last two digits of

$$7^{7^{7^{\cdot^{\cdot^{\cdot^{7}}}}}}$$

in which the number of 7s appearing is 2022? Note $7^7 = 823543$ (or $43 \pmod{100}$), and $7^{7^7} = 7^{823543} \neq (7^7)^7 = 823543^7$.

Problem 2. Prove that if $a, b, c, m \in \mathbb{Z}$, $c \neq 0$, and $ac \equiv bc \pmod{mc}$, then $a \equiv b \pmod{m}$.