PROBLEM 1. When is $a \equiv b \pmod 2$? $a \equiv b \pmod 1$? $a \equiv b \pmod 0$?

PROBLEM 2. Suppose $a \equiv a' \pmod m$ and $b \equiv b' \pmod m$.

(a) Prove that $a + b \equiv a' + b' \pmod m$.

(b) Prove that $ab \equiv a'b' \pmod m$.

PROBLEM 3. Let $n \geq 1$.

(a) Show that if $a \equiv 1 \pmod m$, then $a^n \equiv 1 \pmod m$.

(b) Show that if $a \equiv m - 1 \pmod m$, then $a^n \equiv 1 \pmod m$ if n is even and $a^n \equiv m - 1 \pmod m$ if n is odd.

PROBLEM 4.

(a) Compute the remainder modulo 6 of

$$334 \cdot 545 + 191 \cdot 63.$$

(b) Today is Wednesday. What day will it be 3^{20} days from today?

PROBLEM 5. Recall the equivalence relation from the mini-lecture: having fixed $m \in \mathbb{Z}$, for $a, b \in \mathbb{Z}$, we say $a \sim b$ if $a - b = km$ for some $k \in \mathbb{Z}$. In other words, $a \sim b$ if and only if $a \equiv b \pmod m$. Take $m > 0$, for convenience.

(a) Show that $\sim$ is an equivalence relation directly from the definition.

(b) State the division algorithm for integers a and m, and use it to determine the number of equivalence classes for $\sim$.

PROBLEM 6. (If you have extra time.) Let $V := \{0, 1, \cdots, m-1\}$ for some positive integer m, and fix $a \in V$. Let $G(a, m)$ be the directed graph with vertex set V and with an edge from b to c if $c = b + a \pmod m$. Draw this graph for various a and m, and try to deduce its general structure.