

**MATH 113: DISCRETE STRUCTURES  
HOMEWORK 32**

Given a positive integer  $a$ , denote by  $\overline{a_k a_{k-1} \dots a_1 a_0}$  its digits in decimal expansion. So in particular, we know that  $0 \leq a_i \leq 9$  and

$$a = a_k \cdot 10^k + a_{k-1} \cdot 10^{k-1} + \dots + a_1 \cdot 10 + a_0.$$

*Problem 1.* Prove that  $a$  is divisible by 11 if and only if

$$\sum_{i=0}^k (-1)^i a_i = a_0 - a_1 + \dots + (-1)^k a_k$$

is divisible by 11.

*Problem 2.* Prove that  $a$  is divisible by 7 if and only if 7 divides  $\overline{a_k \dots a_1} - 2a_0$  (i.e., we remove the last digit and subtract twice it from the result).