Given a positive integer a, denote by $\overline{a_k a_{k-1} \cdots a_1 a_0}$ its digits in decimal expansion. So in particular, we know that $0 \leq a_i \leq 9$ and

$$a = a_k \cdot 10^k + a_{k-1} \cdot 10^{k-1} + \cdots + a_1 \cdot 10 + a_0.$$

Today we will explore various divisibility tests.

PROBLEM 1. Suppose $c|a - b$. Prove that $c|a$ if and only if $c|b$.

PROBLEM 2.

(a) Prove that a is divisible by 2 if and only if a_0 is divisible by 2.

(b) Prove that a is divisible by 5 if and only if a_0 is divisible by 5.

(c) Prove that a is divisible by 10 if and only if a_0 is 0.

PROBLEM 3.

(a) Describe and prove a rule of divisibility by 4 based on the digits of a.

(b) Describe and prove a rule of divisibility by 8 based on the digits of a.

PROBLEM 4.

(a) Prove that a is divisible by 3 if and only if $\sum_{i=0}^{k} a_i$ is divisible by 3.

(b) Prove that a is divisible by 9 if and only if $\sum_{i=0}^{k} a_i$ is divisible by 9.