MATH 113: DISCRETE STRUCTURES
HOMEWORK 27

Problem 1. What is the expected value of the number of digits equal to 3 in a 4-digit positive integer? Write your solution as a fraction a/b in lowest terms. The sample space is

$$S = \{a_1a_2a_3a_4 : a_1 \in \{1, 2, \ldots, 9\}, a_2, a_3, a_4 \in \{0, 1, \ldots 9\}\}$$

[Hint: express the relevant random variable as a sum of simpler random variables, and use linearity of expectation.]

Problem 2. Let π be a permutation of $\underline{n}$. The index i is called an *exceedance* of π if $\pi(i) > i$. For instance, using the notation $\pi(1), \pi(2), \ldots, \pi(n)$ for a permutation π, the permutation $\pi = 3, 2, 4, 1$ has exceedance 2 since $\pi(1) = 3 > 1$ and $\pi(3) = 4 > 3$ (but $\pi(2) = 2$ and $\pi(4) = 1 < 4$).

(a) Let X_i be the random variable on the set of permutations of $\underline{n}$ such that $X_i(\pi) = 1$ if i is an exceedance of π, and $X_i(\pi) = 0$, otherwise. What is the expected value, $E(X_i)$?
(b) How many exceedances does the average permutation of $\underline{n}$ have?