MATH 113: DISCRETE STRUCTURES
HOMEWORK DUE WEDNESDAY WEEK 10

Due: Wednesday, April 8 at 10pm.

Problem 1. You have three coins. Two of the coins are fair: when flipped they are equally likely to land heads or tails. One coin, however, is weighted somehow so that its probability of landing heads is $2/3$.

(a) Choose one of the three coins uniformly at random and flip it. What is the probability the result is heads? For your solution, number the coins $1, 2, 3$ with coin 3 being the weighted one, and let A_i denote the event that coin i was chosen. Apply the generalized law of total probability.

(b) Choose one of the three coins at random and flip it. It lands heads. What is the probability that you chose the weighted coin? (Hint: Bayes' law.)

(c) Choose one of the three coins at random and flip it twice. Let E be the event that you get heads on the first flip and F be the even that you get heads in the second flip. Are E and F independent? Prove your answer.

Problem 2. (We reconsider one of the challenge problems from an in-class worksheet.)

Your friend invites you to play a game: they write ten distinct real numbers on ten blank cards. The cards are shuffled randomly and placed face down on the table. You start at the top of the deck and start revealing cards. At any point you may choose to stop turning over cards and select the most recently revealed card. You win if your selection is the largest of all ten numbers (both those previously revealed and those still unrevealed).

It is beyond the scope of this class to prove that the best strategy is given by the *stopping rule*. For the stopping rule, we fix a number r such that $1 \le r \le 10$. We look at the first $r - 1$ cards and note the maximal value among them, M. For the subsequent $10 - (r - 1)$ cards, select the first one larger than M. We will compute the probability of winning when using this strategy. For the remainder of the problem, we will assume $r > 1$ (the case $r = 1$ can be analyzed directly).

(a) For $1 \le a \le 10$, assume that the number in position a is the largest in the deck. What is the probability that we select a with this strategy? (Note that your answer will depend on whether or not $a < r$.)

(b) Use the law of total probability and your answer to (a) to compute the probability that you win using this strategy.

(c) Use a calculator or computer to compute the probabilities of winning with this strategy for all $2 \le r \le 10$. Which r gives you the best chances of winning?