PROBLEM 1 (The Monty Hall problem). A game show provides
contestants with the opportunity to win a car. There are three doors
labeled A, B, and C. Behind two of the doors are goats, and behind
one of the doors is a car. For reasons not completely clear to your
instructor, you hope to select the car instead of a goat. The game
proceeds in the following fashion: First, you select a door. Next, the
host reveals a goat behind one of the remaining doors. (Since there are
two goats, there is at least one goat to reveal.) You are then given the
chance to switch your guess. If your final guess is the door with the
car behind it, you win the car. **Question:** Is it advantageous to switch
your guess?

Here are some assumptions on the problem which should remove
any ambiguity:

- The probability that the car is placed behind any one of the three
  doors is $1/3$.

- The host knows where the car is.

- If the contestant picks a door with a goat behind it at the beginning,
  the host opens the remaining door with a goat before giving the
  option to switch. If the contestant picks the door with the car
  behind it, the host opens any of the other doors with probability
  $1/2$.

Suppose that you initially pick door A and then let A, B, and C
denote the events "the car is behind door A," "door B," and "door C,"
respectively. Let M_A, M_B, and M_C denote the events "the host opens
door A," "door B," and "door C," respectively.

(a) What are $P(M_C|A)$, $P(M_C|B)$, and $P(M_C|C)$?

(b) What is $P(M_C)$? (Use the Law of Total Probability.)

(c) Suppose that the host opens door C revealing a goat. You should
    switch your guess to B if $P(B|M_C) > P(A|M_C)$. Compute these
    conditional probabilities (via Bayes' Law) and draw a conclusion.
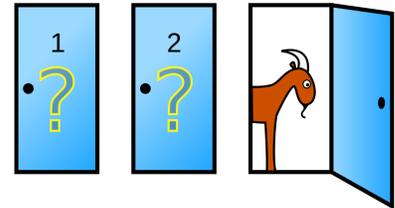
PROBLEM 2. Angélica is taking a true-false test and always marks the
correct answer when she knows it and decides true or false on the ba-
sis of flipping a fair coin when she does not know it. If the probability
that she will know an answer is $3/5$, what is the probability that she
knew the answer to a correctly marked question?