MATH 113: DISCRETE STRUCTURES
HOMEWORK 25

Due: Monday, April 6 at 10pm.

Problem 1. Suppose a fair coin is flipped $n > 1$ times. We record the result at a string of length n in the letters H and T. For instance, if $n = 3$, then HTT says that the first flip was heads and the next two were tails. Our sample space S is, thus, the set of strings of length n consisting of the letters H and T, upon which we place the uniform distribution. Let A be the event that the first flip is heads, and let B be the event that the last flip is heads.

(a) Prove that these events are independent by computing the relevant probabilities and using the definition of independence.

(b) For the case of $n = 2$, give an example of a probability distribution on S for which these same two events A and B are *not* independent. Note that to give a probability distribution you will need to assign probabilities to each of the outcomes, i.e., to each of the elements of S.

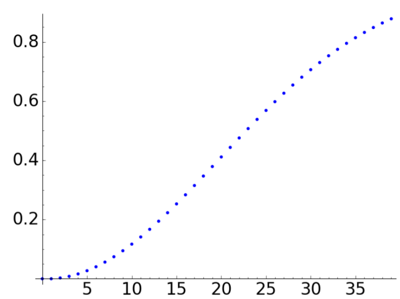
Problem 2. This is the famous birthday problem. Suppose there are n people in a room. For simplicity, we will say that there are 365 possible birthdays (i.e., we will ignore leap years) and that each day is equally likely to be a birthday. To create the sample space, S, number the people from 1 to n, then the possible outcomes are the sequences of length n where the i-th element of the sequence is a possible birthday for person i. Let B be the event that at least two people in the room share a birthday.

(a) What is $|S|$?

(b) Consider the complementary event B^c, i.e., the event that no two people have the same birthday. Give an expression for B^c, and use it to find the probability $P(B^c)$. (The result will depend on n.)

(c) Use the previous result to give an expression for $P(B)$.

(d) A plot of $P(B)$ as a function of n looks like this:



Use a calculator of some sort to give decimal approximations, accurate to three decimal places, for $P(B)$ when $n = 22$ and $n = 23$. You should see that if the room contains at least 23 people, then the probability two people share the same birthday is more than one half.