# MATH 113: DISCRETE STRUCTURES
# HOMEWORK 23

**Due:** Wednesday, April 1 at 10pm.

*Problem* 1. Consider the sequence $a_0, a_1, a_2, \ldots$ determined by

$$a_0 = 0 \quad \text{and} \quad a_{n+1} = 2a_n + 1 \text{ for } n \geq 0.$$

(a) Compute the first few members of the sequence. Make a conjecture for a closed formula for the sequence and prove it using induction.
(b) Let $A(x) = a_0 + a_1 x + a_2 x^2 + \ldots$ be the generating function for this sequence. Find a closed formula for $A(x)$ using the recurrence for $(a_n)_{n=0}^{\infty}$.
(c) Use your answer to (b) to confirm the closed formula you found in (a).

*Problem* 2. Recall that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$, and recall the definition of multiplication of power series.
(a) What sequence corresponds to $\frac{1}{(1-x)^2}$?
(b) What sequence corresponds to $\frac{1}{(1-x)^3}$?

*Problem* 3. (BONUS) Recall the recurrence for the sequence of Catalan numbers:

$$C_0 = 1 \quad \text{and} \quad C_{n+1} = \sum_{k=0}^{n} C_k C_{n-k} \text{ for } n \geq 0.$$

Find a closed formula for the generating function for the Catalan sequence.