

PROBLEM 1. Suppose that $a \in \mathbb{R}^{\mathbb{N}}$ is a polynomial sequence of degree 4. Use the following table of differences to determine a formula for a_n .

a_n	0	0	4	12	72	...
$\Delta[a]_n$		0	4	8	60	...
$\Delta^2[a]_n$			4	4	52	...
$\Delta^3[a]_n$				0	48	...
$\Delta^4[a]_n$					48	...

PROBLEM 2. With your group, choose a “random” polynomial p of degree at most 3. Prepare a table of the values $p(n)$ for $n = 0, 1, 2, 3$. Swap tables of values with another group and then reconstruct each others polynomials. Here is an example of <https://sagecell.sagemath.org/> code that might help:

```
f(x) = 5*x^3 + 2*x^2 + 10*x + 6
[f(i) for i in range(5)]
```

Output:

```
[6, 23, 74, 189]
```

PROBLEM 3.

- (a) Fix $r \geq 0$. For $n \geq 0$ define the sequence whose n -th term is $a_n = \sum_{k=0}^n k^r$. Prove that $(a_n)_{n=0}^{\infty}$ is a degree $r + 1$ polynomial sequence. (Hint: our reading implies that it suffices to show that $\Delta[a]_n$ is a polynomial sequence of degree r .)
- (b) Consider the case $r = 3$. Use a table of differences to determine a polynomial expression for

$$a_n = \sum_{k=0}^n k^3.$$

Fun fact: you can write that polynomial as the square of a single binomial coefficient involving n .

PROBLEM 4. Prove that 3 divides $n^3 + 2n$ for all $n \in \mathbb{N}$ by using a table of differences to write $\frac{1}{3}n^3 + \frac{2}{3}n$ as a sum of $\binom{n}{k}$ s.