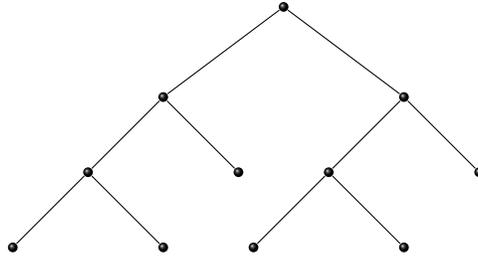# MATH 113: DISCRETE STRUCTURES
HOMEWORK 21

Due: Friday, March 13 at 10pm.

Problem 1. Consider the full binary tree T:

Using the bijections described in our text, find each of the following Catalan structures corresponding to T:

(a) a full parenthesization of the letters a, b, c, d, e, f ("full" means each multiplication is binary, i.e., involves two factors);
(b) a balanced parenthetical expression with five pairs of ()s;
(c) and a Dyck path of length ten.

Note: It is important to precisely use the conventions used in the text. For example, reordering labels, etc., will not yield the same bijection.

Problem 2. Coin stackings form another set of Catalan structures (i.e., their count is given by Catalan numbers). Here are the $C_3 = 5$ coin stackings with a base of three coins:
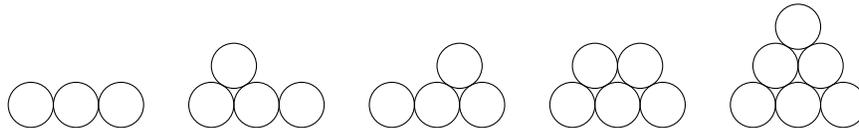
(a) Draw the $C_4 = 14$ coin stackings with a base of 4 coins.
(b) Prove that the number of coin stackings with a base of n coins is C_n by describing a bijection between them and Dyck paths of length $2n$. [Hint: consider the region between a Dyck path and the diagonal.]