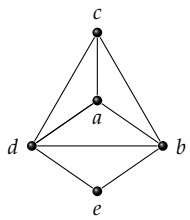
PROBLEM 1. Consider the graph pictured in the margin.



(a) Find a path of maximal length. (Recall: a path contains no repeated vertices.)

(b) Find a cycle containing all of the vertices.

(c) Find an Eulerian walk from a to c.

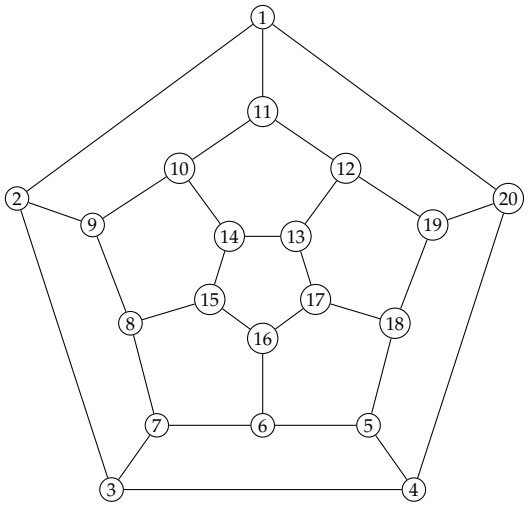
(d) Find a Hamiltonian cycle.

PROBLEM 2. Let G be a graph.

(a) Let u and v be two vertices of G. Prove that if there is a walk in G from u to v, then there is a path in G from u to v.

(b) Define a relation on the set V of vertices of G as follows: $u \sim v$ if there exists a path in G from u to v. Prove that $\sim$ is an equivalence relation on V.

PROBLEM 3. Does the dodecahedron graph have a Hamiltonian cycle? If so, demonstrate one by listing its vertices.



PROBLEM 4. How many Hamiltonian paths are there in a complete graph? How many Hamiltonian cycles?

PROBLEM 5. Does every bipartite graph have a Hamiltonian cycle?