PROBLEM 1. Consider the following relations on the set $\mathbb{R}$ of real numbers: inequality ($\neq$), strictly greater than ($>$), and less than or equal to ($\leq$). Determine which (if any) of the three properties of an equivalence relation these relations have:

relation	reflexivity	symmetry	transitivity
$\neq$			
$>$			
$\leq$			

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Template for proving a relation is an equivalence relation.

Theorem. Define a relation $\sim$ on a set A by blah, blah, blah. Then $\sim$ is an equivalence relation.

Proof. *Reflexivity.* For each $a \in A$, we have $a \sim a$ since blah, blah, blah. Therefore, $\sim$ is reflexive.
Symmetry. Suppose that $a \sim b$. Then, blah, blah, blah. It follows that $b \sim a$. Therefore $\sim$ is symmetric.
Transitivity. Suppose that $a \sim b$ and $b \sim c$. Since blah, blah, blah, it follows that $a \sim c$. Therefore, $\sim$ is transitive.
Since $\sim$ is reflexive, symmetric, and transitive, it follows that $\sim$ is an equivalence relation. □

PROBLEM 2. Consider the relation $\sim$ on $\mathbb{R}$ such that $x \sim y$ if and only if $x - y$ is an integer.

Recall that for $\simeq$ an equivalence relation on set X, $X/\simeq$ is the set of equivalence classes for $\simeq$.

(a) Give a formal proof (following our template) that $\sim$ is an equivalence relation.

(b) Draw the real number line, choose a point, and draw that point's equivalence class. Repeat for several points.

(c) What does a generic element of $\mathbb{R}/\sim$ look like? Does $\mathbb{R}/\sim$ has a natural "shape"?

PROBLEM 3. We place two red and two black checkers on the corners of a square. Say that two configurations are equivalent if one can be rotated to the other.

(a) Check that this is an equivalence relation.

(b) Draw the elements in each equivalence class.

(c) If $\sim$ is a relation on a finite set S, and each equivalence class has the same number k of elements, then the overcounting principle says the number of equivalence classes is $|S|/k$. Why don't these ideas apply to our problem?

PROBLEM 4. A total of n Terraneans and n Gethenians* attend a meeting and sit around a round table. If Terraneans and Gethenians alternate seats, in how many ways may they be seated up to rotation? Discuss your solution in terms of an equivalence relation and equal-sized equivalence classes.

* From the planet Gethen, which is the setting of *The Left Hand of Darkness* by Ursula K. Le Guin.