# MATH 113: DISCRETE STRUCTURES
# HOMEWORK 06

**Due:** Wednesday, February 11.

*Problem* 1. Let $A, B$ be nonempty sets and let $f: A \to B$ be a function. Show that a function $g: B \to A$ such that $g \circ f = \mathrm{id}_A$ exists if and only if $f$ is injective. (Note that this is an "if and only if" proof. So there will be two parts to your proof: first suppose there is a function $g$ with the stated properties, and show that it follows that $f$ is injective; next, suppose that $f$ is injective, and use that to prove that the appropriate function $g$ exists. For this last part you need to construct the function $g$.)

*Problem* 2. Let $A$ be a nonempty finite set, let $E \subseteq 2^A$ be the collection of subsets of $A$ of even cardinality, and let $O \subseteq 2^A$ be the collection of subsets of $A$ of odd cardinality. Create an explicit bijective function $f: E \to O$ and conclude that $|E| = |O| = 2^{|A|-1}$. (You should define $f$ by giving an explicit procedure one can perform to turn an element of $E$ into an element of $O$. You should prove that $f$ is bijective either by exhibiting a two-sided inverse, or by proving that $f$ is injective and surjective.)