General instructions

(a) Write down full, detailed solutions to problems 1 and 2.

(b) Exchange your solutions with your neighbor.

(c) Read their solutions carefully. Take note of things that were confusing on a first read, think about suggestions for improvement. Also take note of things that were effective.

(d) Have a conversation with your neighbor about your solutions and theirs.

(e) Read and critique with your neighbor the solutions presented below for problem 3.

PROBLEM 1. To form a password, you can either form as sequence of six digits from $\{0, 1, \ldots, 9\}$ or a sequence of four letters from $\{a, \ldots, z\}$.

(a) How many possible passwords are there if no number or letter can be repeated?

(b) How many if repetitions are allowed?

PROBLEM 2. Let A, B, C be sets. Prove that

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

PROBLEM 3. There are 12 students. How many ways can I pair you up? This means breaking up the whole class into groups of two.

SOLUTION:
$$\frac{12!}{2^6 \cdot 6!}$$
Since there are first 12 choices, then 11, and so on down to 1 choices, that gets all of the total possible orderings of students. This doesn't account for overcounting due to pairs being two ways, and the orderings of pairs not mattering. To fix this we divide by the number of possible ways that the same pairs can give us different reorderings, which is 6! and also by the amount of ways that variation within the groups can increase the amount of possible results, which is 2^6. Since in pairs there are only two options.

SOLUTION: First pick 2 out of the 12 students, then 2 out of the remaining 10, then so on until there are 6 since it is the total number of students divided by the size of each pair. As the order does not matter we can divide the number of ways the 6 pairs can be arranged.

SOLUTION: With 12 students you can pair up the class: $(11 \cdot 9 \cdot 7 \cdot 5 \cdot 3 \cdot 1)$ ways.

SOLUTION: I thought of this problem as a tree. The 1st student has 11 choices, all but themself. The next student will have 9 choices, since the 1st student and their partner are not longer a choice. This continues until there are only two students, and no choice, left, resulting in this solution:
$$11 \cdot 9 \cdot \ldots \cdot 1 = 10395.$$

SOLUTION: We can solve this problem by purposefully over-counting and correcting it later. One way to pair students up is by listing all of them in some order, and then pairing the first two students, the next two, and so on. With 12 students, the amount of ways we can do this is 12!, as that is how many reorderings exist of 12 students. However, within each pair, we could switch the order of the students and get the same result, so we over-counted by a factor of 2 for each pair; since there are $\frac{12}{2} = 6$ pairs, we need to divide by 2^6. Finally, we can also rearrange the order of the pairs within the entire set of students; with 6 pairs, this means we over-counted by the amount of ways we can order 6 objects, which is 6!. Putting this all together, our amount of pairs is
$$\frac{12!}{2^6 \cdot 6!} = 10395.$$