PROBLEM 1. Recall that $\mathbb{N} = \{0, 1, 2, \dots\}$ denotes the set of nonnegative integers. Consider the following sets:

$$A = \{x \in \mathbb{Z} \mid x^2 \in \mathbb{N}\},$$
$$B = \{x \in \mathbb{N} \mid x \text{ is even}\} \cap \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\},$$
$$C = \{x \in \mathbb{N} \mid x \text{ is even}\} \cup \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\},$$
$$D = \{x \in \mathbb{N} \mid x \text{ is even}\} \triangle \{x \in \mathbb{N} \mid x \text{ is a multiple of } 3\}.$$

Write out some elements of each set and then describe the set in words, justifying your answer.

PROBLEM 2. Recall that De Morgan's law states that for all sets A, B, C,

$$C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$$

and

$$C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B).$$

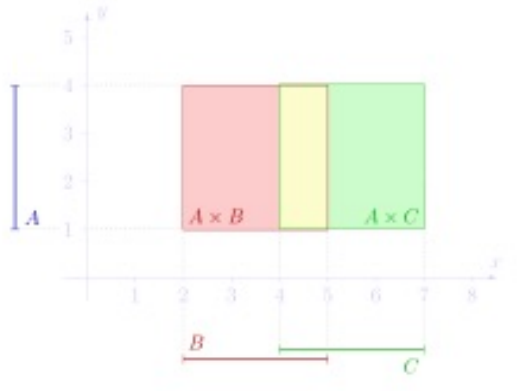
(a) Draw Venn diagrams that express these identities.

(b) Prove the second identity.

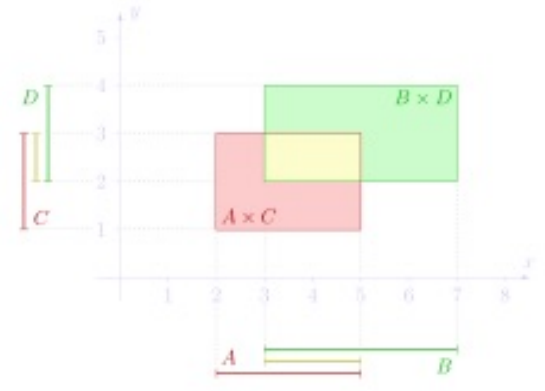
In order to prove an equality of sets $X = Y$, you can show $X \subseteq Y$ and $Y \subseteq X$.

PROBLEM 3. Suppose that A and B are finite sets with $|A| = m$, $|B| = n$, and $m \leq n$. What are the smallest and largest possible values of $|A \cap B|$?

PROBLEM 4. Explain how the following pictures illustrate the indicated identities, and then prove one or both of them.



$A \times (B \cap C) = (A \times B) \cap (A \times C)$



$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$