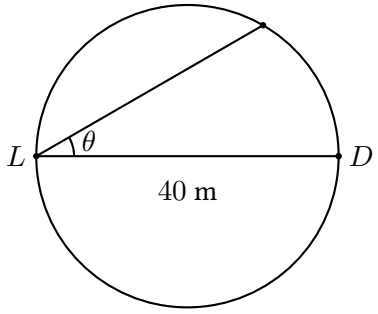
MATH 111: CALCULUS
HOMEWORK DUE FRIDAY WEEK 9

Make sure to review the homework instructions in the syllabus before writing your solutions. In particular, show your work and write in complete sentences (but also aim for concise explanations).

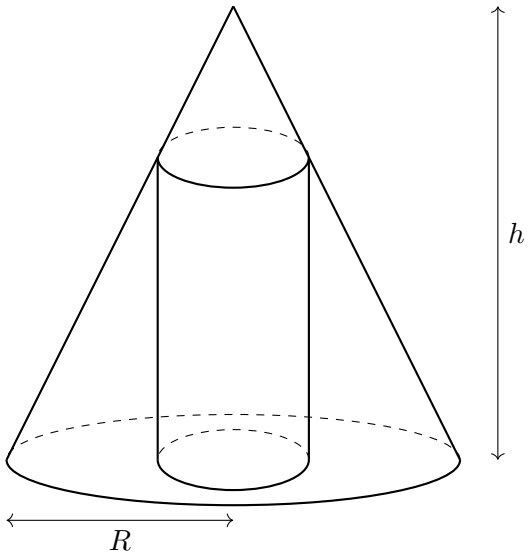
Problem 1. Suppose that g is a differentiable function and $g'(2) = 0$. In addition, suppose that on $1 < x < 2$ and $2 < x < 3$, it is known that $g'(x)$ is positive.

(a) Does g have a local maximum, local minimum, or neither at $x = 2$? Why?
(b) Suppose that $g''(x)$ exists for every x such that $1 < x < 3$. Reasoning graphically, describe the behavior of $g''(x)$ for x-values near 2.
(c) Besides being a critical number of g, what is special about the value $x = 2$ in terms of the behavior of the graph of g?

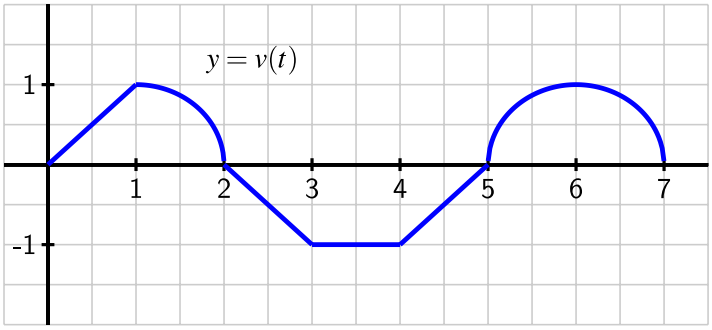
Problem 2. Consider a lifeguard L at a circular pool with diameter 40m. They must reach someone who is drowning at the exact opposite side of the pool (position D in the diagram). The lifeguard swims with a speed v and runs around the pool with a speed $3v$. If the lifeguard first swims at angle θ across the pool (as indicated in the diagram) and then runs to D around the edge of the pool, at what angle θ should they swim in order to minimize the time of their journey.



Problem 3. Find the largest volume of a cylinder that fits into a cone that has base radius R and height h as indicated in the diagram on the next page.



Problem 4. An object moving along a horizontal axis has its instantaneous velocity at time t in seconds given by the function v pictured below, where v is measured in feet per second. Assume that the curves that make up the parts of the graph of $y = v(t)$ are either portion of straight lines or portions of circles.



(a) Determine the exact total distance the object traveled on $0 \leq t \leq 2$.
(b) What is the value and meaning of $s(5) - s(2)$, where $y = s(t)$ is the position function of the moving object?
(c) On which time interval did the object travel the greatest distance: $[0, 2]$, $[2, 4]$, or $[5, 7]$?
(d) On which time interval(s) is the position function $s$increasing? At which point(s) does s achieve a relative maximum?