MATH 111: CALCULUS
HOMEWORK DUE FRIDAY WEEK 3

Make sure to review the homework instructions in the syllabus before writing your solutions. In particular, show your work and write in complete sentences (but also aim for concise explanations).

Problem 1. Consider the function

$$g(x) = |x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0, \end{cases}$$

commonly referred to as the *absolute value* function.

(a) Use the central difference approximation

$$g'(0) \approx \frac{g(0+h) - g(0-h)}{2h}$$

to approximate $g'(0)$. Do we get different approximations for different values of h?

(b) Now approximate $g'(0)$ using the backward and forward differences

$$\frac{g(0) - g(0-h)}{h}, \qquad \frac{g(0+h) - g(0)}{h}$$

for some (any?) small positive h.

(c) Explain why the limit definition of differentiation implies that $g'(0)$ does not exist. Explain how this is possible despite your answer to (a).

(d) (Optional, but worthwhile.) Fix a positive real number m. Find a function $f(x)$ for which central difference approximation to $f'(0)$ always equals m, yet $f'(0)$ does not exist.

Problem 2. Consider the function

$$h(x) = \frac{\sin x}{x}.$$

(a) Use desmos to sketch the graph of $y = h(x)$.

(b) What does your graph indicate about the values of

$$\lim_{x \to \infty} h(x) \qquad \text{and} \qquad \lim_{x \to -\infty} h(x)?$$

(c) What does your graph indicate about the value of

$$\lim_{x \to 0} h(x)?$$

(d) Based on (c), what should the value of $\sin'(0)$ be, and what is the equation of the tangent line to $y = \sin(x)$ at $x = 0$?

(e) Use your answer to (d) to approximate the value of $\sin(0.001)$ without using a calculator.

Problem 3. The value (in US dollars), V, of a particular car depends on the number of miles m, the car has been driven, according to the function $V = d(m)$.

(a) Suppose that $d(40000) = 15500$ and $d(55000) = 13200$. What is the average rate of change of d on the interval $[40000, 55000]$, and what are the units of this value?

(b) In addition to the information given in (a), say that $d(70000) = 11100$. Determine the best possible estimate of $d'(55000)$ and write one sentence to explain the meaning of your result, including units on your answer.

(c) Which value do you expect to be greater: $d'(30000)$ or $d'(80000)$? Why?

(d) There's an old adage that a car loses 10% of its value when you drive it off the lot (*i.e.*, immediately after purchase). If you purchase the car when $m = 100$, what does this tell you about $d'(100)$?

(e) Draw a plausible sketch of $V = d(m)$ incorporating all of the above information and assuming the purchase price of the car is \$25000. Write a sentence to describe the long-term behavior of $V = d(m)$, plus another sentence to descrie the long-term behavior of $d'(m)$. Your discussion should be in practical terms regarding the value of the car and the rate at which the value is changing.