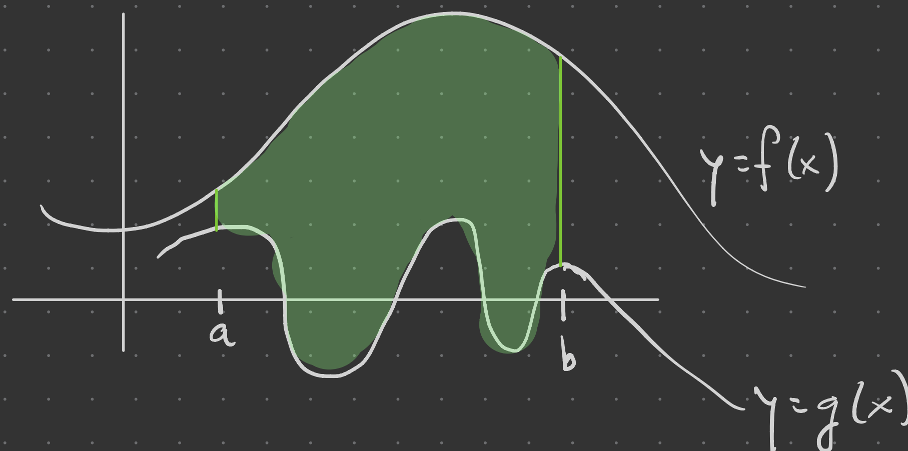
24 . XI . 13

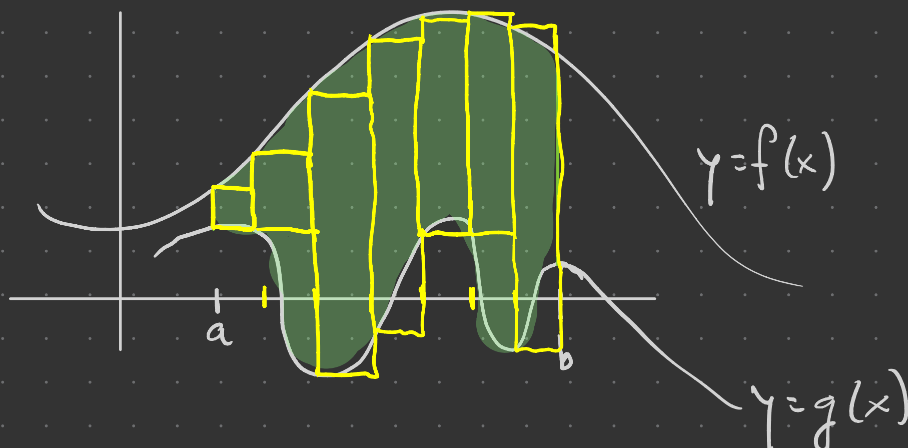
Goals · Area between curves

Suppose $f(x) \geq g(x)$ for x in $[a,b]$. What is the area of the region $\{(x,y) \mid a \leq x \leq b,\ g(x) \leq y \leq f(x)\}$?



We can approximate the area with a Riemann sum,

$$\sum_{i=1}^{n} \left(f(x_i^*) - g(x_i^*)\right) \Delta x$$

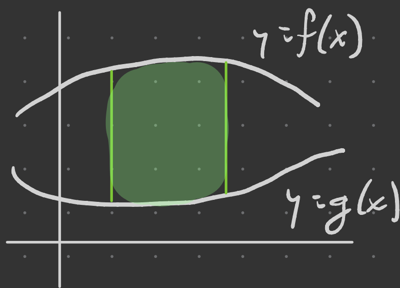


As $n \longrightarrow \infty$, the approximation converges to the area and

$$A = \int_a^b (f(x) - g(x)) \, dx \,.$$ [for $f(x) \geq g(x)$]

Note This is equivalent to

$$A = \int_a^b f(x) \, dx - \int_a^b g(x) \, dx \,.$$



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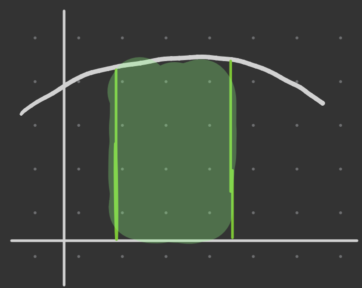
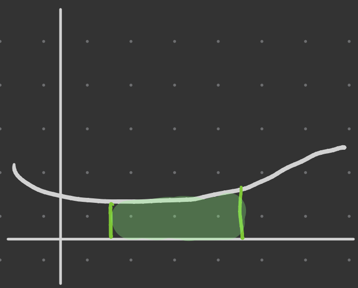
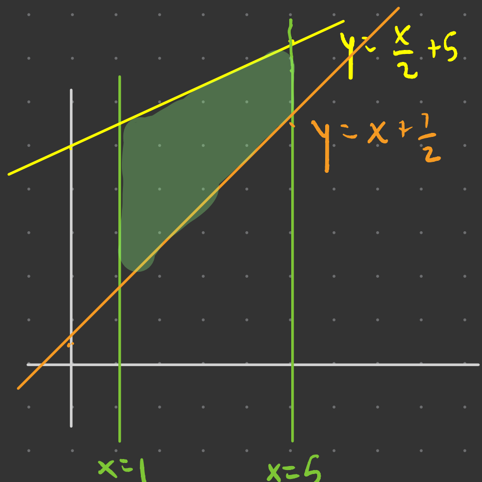




Question Does this still make sense when one or both functions are negative?

E.g. What is the area of the region bounded by

$x=1$, $x=5$, $y=\frac{x}{2}+5$, and $y=x+\frac{1}{2}$?



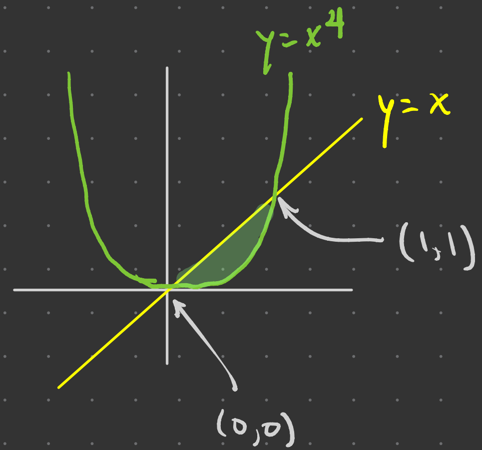
$$A=\int_1^5\left(\frac{x}{2}+5-\left(x+\frac{1}{2}\right)\right)dx$$

$$=\int_1^5\left(-\frac{1}{2}x+\frac{9}{2}\right)dx$$

$$=\left(-\frac{1}{4}x^2+\frac{9}{2}x\right)\Big|_1^5$$

$$=12$$

E.g. What is the area of the region below $y=x$ and above $y=x^4$?



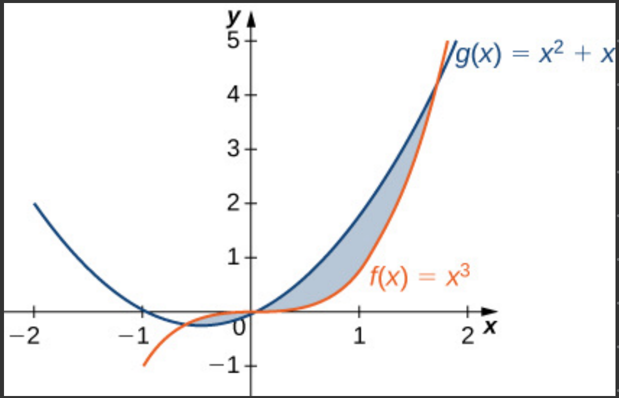
$$A=\int_0^1 (x-x^4)\,dx$$

$$=\frac{x^2}{2}-\frac{x^5}{5}\Big|_0^1$$

$$=\frac{1}{2}-\frac{1}{5}=\frac{3}{10}.$$

Problems Find the following areas :

(1)



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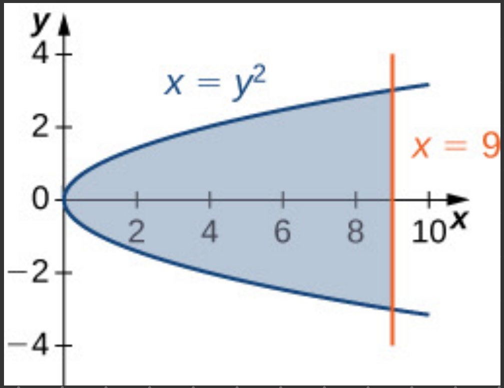

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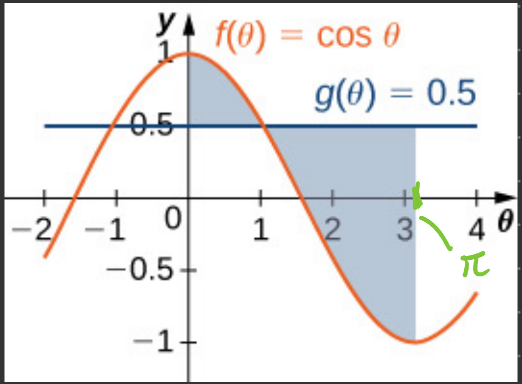

(4)