$$= e^{-1/x^2} \cdot (2x^{-3})$$

$$= \frac{2e^{-1/x^2}}{x^3}.$$

Later! ☺ 24.X.2

Recall $\log_b = B^{-1}$ for $B(x) = b^x$.

Let $\log_e = \exp^{-1} =: \log$ (also denoted $\ln$).

Thm $(\log x)' = \frac{1}{x}$ for $x > 0$.

Pf By the inverse function theorem,

$$(\log x)' = \frac{1}{\exp(\log x)}$$

$$= \frac{1}{x}. \quad \square$$

After learning about implicit differentiation, we will show

$$(\log_b x)' = \frac{1}{(\log b)x},$$

$$(b^x)' = (\log b)\, b^x.$$

$x \in A$
"x is in A"

Optimization

"absolute"

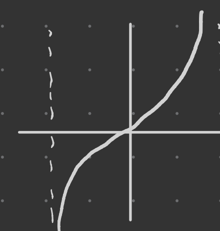
Let I be an interval and $f: I \to \mathbb{R}$ a function.

Defn We say f has a global maximum on I at c when $f(c) \geq f(x)$ for all x in I. We say it has a global minimum on I at c when $f(c) \leq f(x)$ for all x in I. A global max/min is called a global extremum.

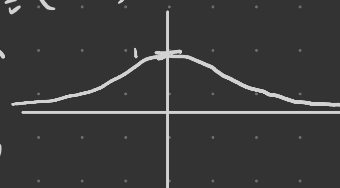
E.g.

- $f(x) = \tan x$ on $(-\pi/2, \pi/2)$

  <u>no</u> global extrema



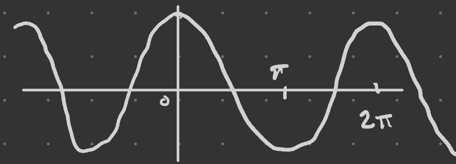
- $g(x) = \dfrac{1}{1+x^2}$ on $\mathbb{R} = (-\infty, \infty)$



Global max at $x = 0$

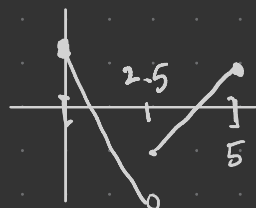
<u>No</u> global min.

- $h(x) = \cos x$ on $\mathbb{R}$



Global maxima at $x = 0, \pm 2\pi, \pm 4\pi, \ldots$

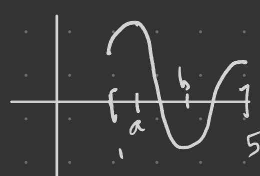
Global minima at $x = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

- 

on $[0,5]$

Global max at $x = 0$

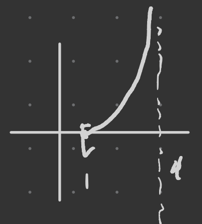
<u>No</u> global min

- 

on $[1,5]$

Global max at $x = a$

Global min at $x = b$



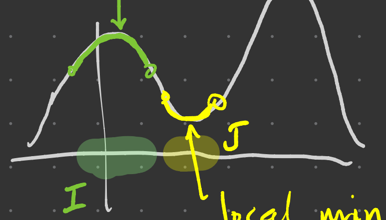
on $[1, 4)$

Global min at $x = 1$

No global max

Extreme Value Theorem If f is a continuous function on a closed bounded interval $[a, b]$, then f has a global max and a global min on $[a, b]$.

Local extrema



not a global max, but a local one

Defn A function f has a local max/min at c if there exists an open interval I containing c on which f has a global max/min at c.

These poits are called local extrema.

Defn Let c be an interior point in the domain of f. Call c a critical point of f when $f'(c)=0$ or $f'(c)$ undefined.

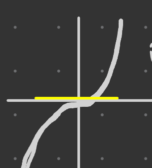
∴ { horizontal or undefined tangent line at c

[Fermat]

Thm If f has a local extremum at c, then c is a critical point of f.

critical point $\not\Rightarrow$ local extremum.

Consider $f(x)=x^3$ at $x=0$:



$f'(x)=(x^3)'=3x^2$

$f'(0)=3\cdot 0^2=0$

<u>Pf of Thm</u> Suppose f has a local extremum at c. We need to show that f diff'l at $c \Rightarrow f'(c)=0$.

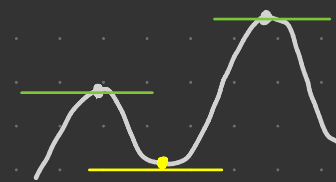
Assume f has a local max at c (local min case similar). Take an open interval I containing c on which $f(c) \geq f(x)$ for all $x \in I$.

Since f diff'l at c, $f'(c) = \lim_{x \to c} \dfrac{f(x)-f(c)}{x-c}$ exists.

For $x \leq c$ near c and in I, $f(x) \leq f(c)$ so $f(x)-f(c) \leq 0$ and $x-c<0$ so $\dfrac{f(x)-f(c)}{x-c} \underset{1}{\geq} 0$.

For $x \geq c$ near c and in I, still have $f(x)-f(c) \leq 0$

but $x-c>0$ so $\dfrac{f(x)-f(c)}{x-c} \leq 0$ $\quad$ (2)



By (1), $f'(c) = \lim\limits_{x \to c^-} \dfrac{f(x)-f(c)}{x-c} \geq 0$.

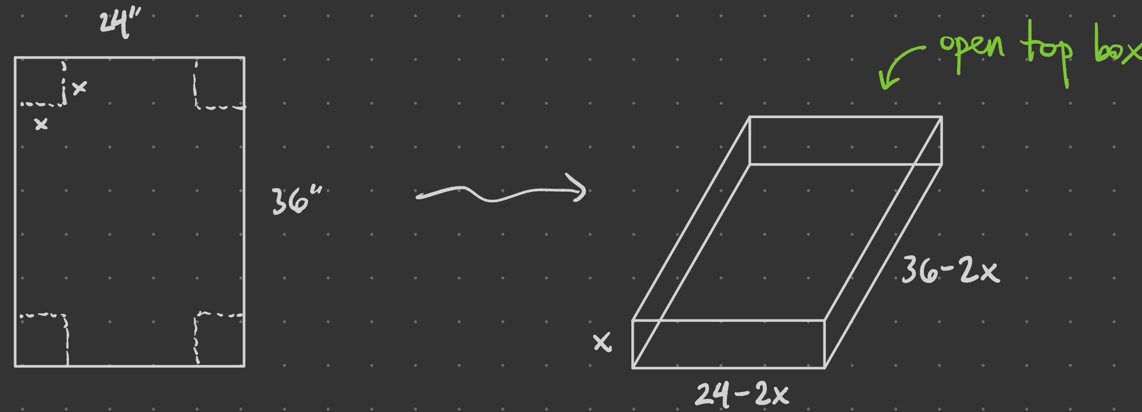
By (2), $f'(c) = \lim\limits_{x \to c^+} \dfrac{f(x)-f(c)}{x-c} \leq 0$.

Since $0 \leq f'(c) \leq 0$, $f'(c) = 0$. $\square$

$f'(c) = 0 \not\Rightarrow f$ has a local extremum at c.

To find absolute extrema: compare values at crit pts + endpts.

E.g. Construct a box as follows:



What choice of x maximizes the volume of the box?

$$V = x(24-2x)(36-2x) = 4x^3 - 120x^2 + 864x \quad \text{in}^3$$

Need to maximize V over $[0, 12]$.

Know max at $x=0$, $x=12$, or a critical pt c where $\boxed{V'(c)=0}$ (or $V'(c)$ undefined).

won't happen as V is polynomial

Compute $V'(x) = 12x^2 - 240x + 864$

$$= 12(x^2 - 20x + 72)$$

$$12 \overline{)864} \quad 72$$
84
24

Find c such that $0 = V'(c) = 12(c^2 - 20c + 72)$

Via quadratic fmla,

$$c = \frac{20 \pm \sqrt{(-20)^2 - 4 \cdot 1 \cdot 72}}{2 \cdot 1}$$

$$= \frac{20 \pm \sqrt{112}}{2} = 10 \pm \frac{1}{2}\sqrt{112}$$

$$\begin{array}{r} 72 \\ \times\ 4 \\ \hline 288 \end{array}$$

$\approx \cancel{15.3}$ or 4.7

not in $[0, 12]$

unique crit pt in $[0, 12]$.