24. X. 4

Applied optimization

1. Draw & label
2. Determine quantity to optimize and its domain
3. Find a formula for this quantity
4. Use constraints to make the target function depend on only one variable
5. Find extrema by evaluating at critical and endpoints

Eg. Use 4 feet of wire to form a square and a circle. How much wire should be used for the square to maximize enclosed area of both shapes?

1. 

2. A = total area with $0 \le 4x \le 4$ and $4 = 4x + 2\pi r$ ☆

3. $A = x^2 + \pi r^2$

4. By ☆, $r = \frac{1}{2\pi}(4-4x) = \frac{2(1-x)}{\pi}$

so $A = x^2 + \pi\left(\dfrac{2(1-x)}{\pi}\right)^2 = \dfrac{1}{\pi}\left((\pi+4)x^2 - 8x + 4\right)$.

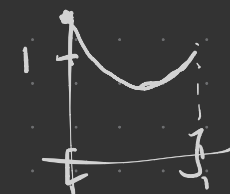
saw $0 \le 4x \le 4 \Rightarrow 0 \le x \le 1$

5 Need to maximize $A(x)$ over $[0,1]$.

$$A'(x) = \frac{1}{\pi}\left(2(\pi+4)x - 8\right) = \frac{2}{\pi}\left((\pi+4)x - 4\right)$$ (defined everywhere)

which satisfies $A'(x) = 0 \iff (\pi+4)x = 4$

$$\iff x = \frac{4}{\pi+4} \approx 0.56.$$



Now check values at critical & endpts:

$A(0) = \dfrac{4}{\pi} \approx 1.273$

$A\left(\dfrac{4}{\pi+4}\right) = \dfrac{4}{\pi+4} \approx 0.56$

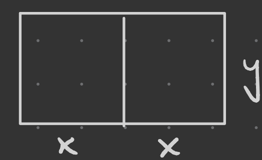
$A(1) = 1$

Thus A attains its maximum when we use all of the wire on the circle!

Note Minimum area with square of side length $\dfrac{4}{\pi+4}$.

Problem 1 Which points of the graph $y = 4 - x^2$ are closest to the point $(0,2)$?

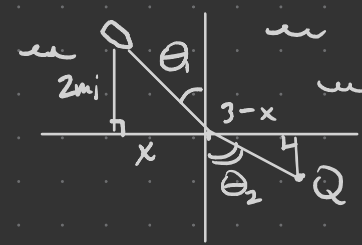
Problem 2 Use 200 ft of fencing to enclose two adjacent rectangular corrals with same dimensions. What dimensions should be used to maximize enclosed area?



Problem 3 Find the dimensions of a window with shape

and perimeter 10 ft with largest area.

Problem 4 A woman is in a boat 2mi from the coast and needs to go to point Q 3mi down the coast, 1mi inland. If she can row



at 2 mph and walk at 4 mph, toward what point on the coast should she row in order to reach Q the fastest?