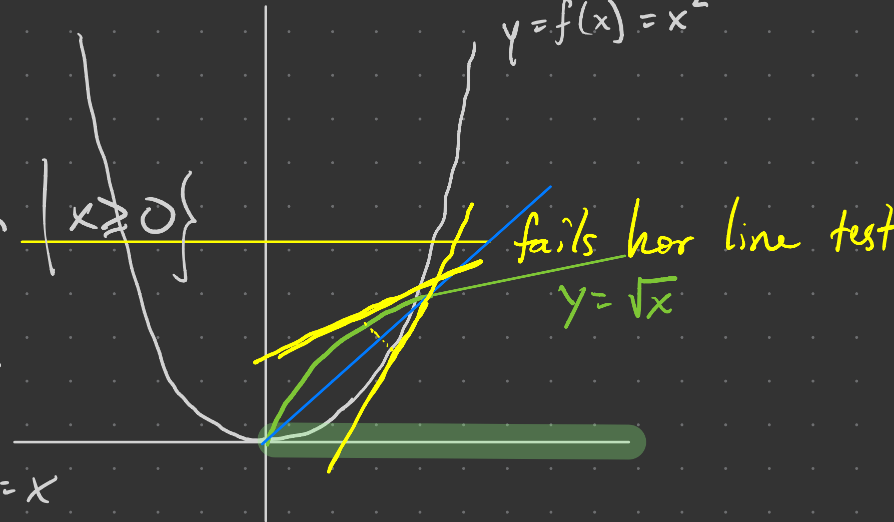
E.g. $f(x) = x^2$

Restrict the domain of f to $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} \mid x \geq 0\}$

$g(x) = \sqrt{x}$ on $\mathbb{R}_{\geq 0}$.

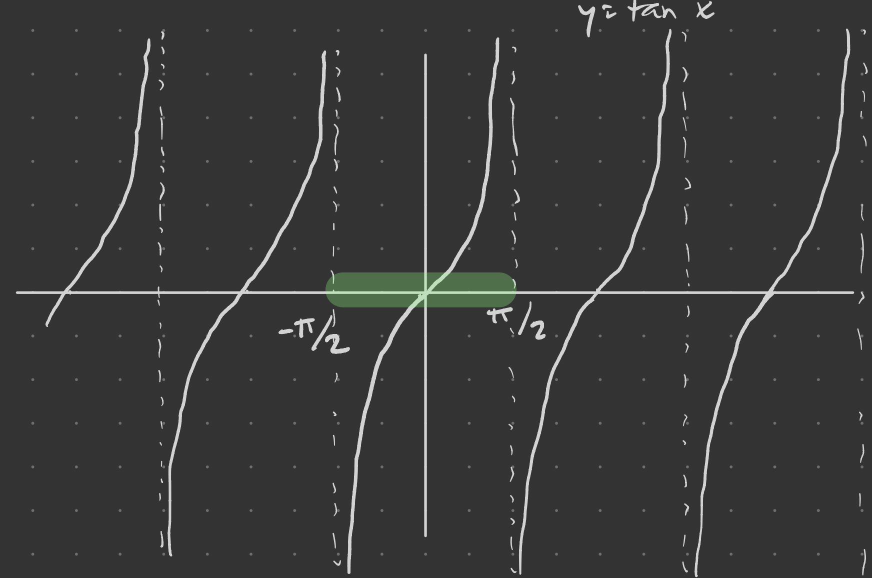


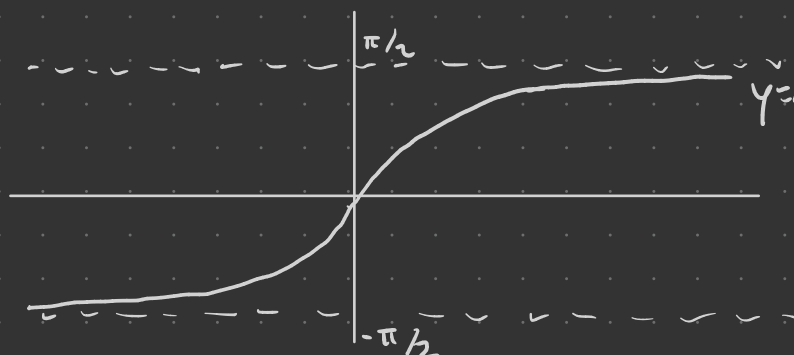
Then $f(g(x)) = (\sqrt{x})^2 = x$

$g(f(x)) = \sqrt{x^2} = x$

Note Graph of $y = f^{-1}(x)$ is the reflection of $y = f(x)$ over $y = x$ line.

E.g. $f(x) = \tan x$



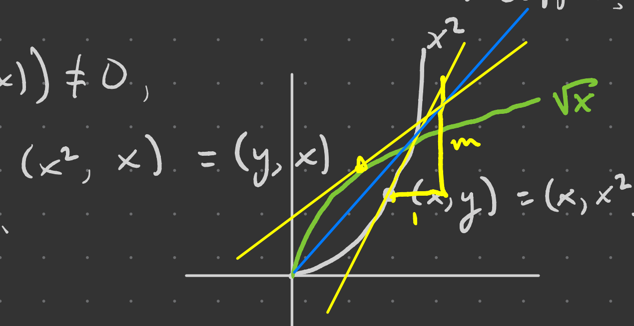


Write arctan for $\tan^{-1}$. Note need to choose region where hor line test hold

Inverse Function Theorem Suppose f is both invertible and diff'l.

Then for all x such that $f'(f^{-1}(x)) \neq 0$,

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$



Pf We know $x = f(f^{-1}(x))$. Differentiating both sides,

$$1 = f'(f^{-1}(x)) \cdot (f^{-1})'(x)$$

$$\Rightarrow (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

[divide by $f'(f^{-1}(x))$]

E.g. If $f(x) = x^2$ then for $x \geq 0$, $f^{-1}(x) = \sqrt{x}$.

Thus $$(\sqrt{x})' = \frac{1}{f'(\sqrt{x})} = \frac{1}{2\sqrt{x}}.$$

Similarly, for all x, $$\underset{\displaystyle x^{1/3}}{\overset{}{(\sqrt[3]{x})'}} = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3}x^{-2/3}.$$

In fact, this is how we extend the power rule to all rational #s!

i.e. $$(x^{1/3})' = \frac{1}{3}x^{1/3-1} = \frac{1}{3}x^{-2/3}$$

Q What about irrational exponents? $\frac{d}{dx}(x^{\pi}) = \pi x^{\pi-1}$ ✓

Rates of change

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \text{instantaneous rate of change of } f \text{ at } x$$

If $s(t)$ = position along a line at time t

then $s'(t) =: v(t)$ = velocity at time t, $|v(t)|$ = speed

$s''(t) = v'(t) =: a(t)$ = acceleration at time t

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2}$$

Units: • If $s(t)$ in meters, t in seconds, then $\frac{s(t+h)-s(t)}{h}$ in $\frac{\text{meters}}{\text{second}} = \frac{m}{s}$ and so is $v(t)$.

- Then $a(t)$ is the limit of $\dfrac{v(t+h)-v(t)}{h}$ which is in

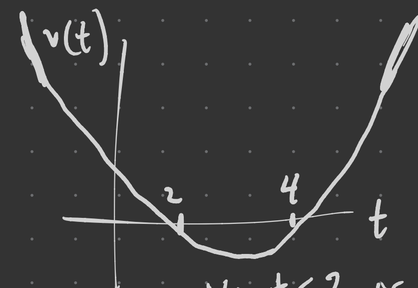
$$\frac{m/s}{s} = \frac{m}{s^2} = \frac{\text{meters}}{\text{second}^2}$$

Problem Position $s(t) = t^3 - 9t^2 + 24t + 4$ meters north of current location t seconds from now.

(a) Find $v(t)$

(b) When is the object at rest?

(c) When is it moving north? south?



N: $t<2$ or $t>4$

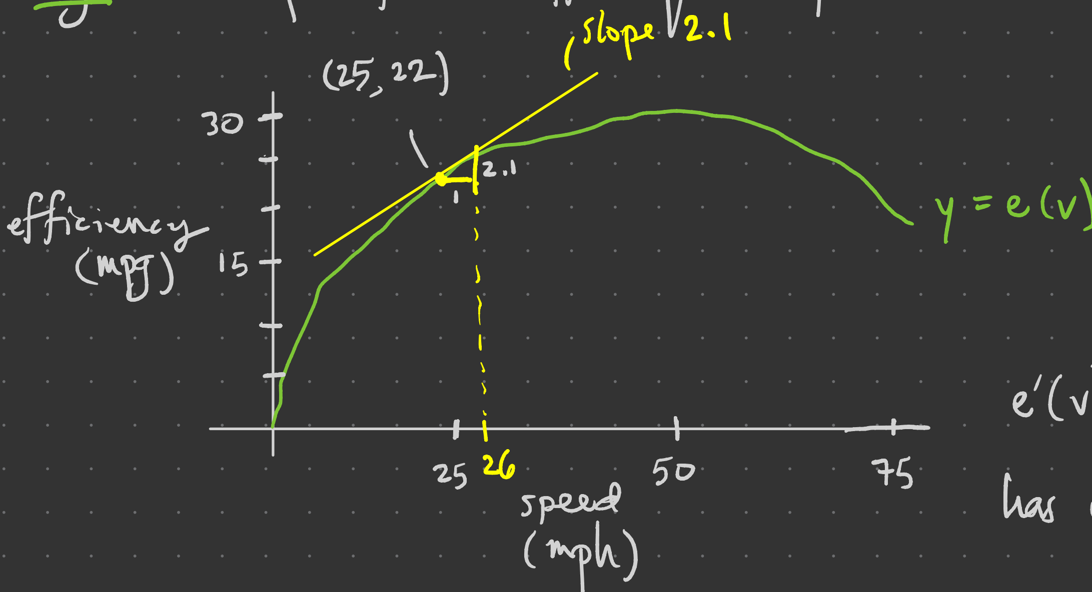
S: $2<t<4$

(a) $v(t) = s'(t) = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8)$

(b) $v(t) = 0 \iff 0 = 3(t^2 - 6t + 8) = 3(t-2)(t-4)$

So at rest for $t = 2$ or 4 seconds

E.g. Graph of fuel efficiency vs speed:



$$e'(v) = \lim_{h \to 0} \frac{e(v+h) - e(v)}{h}$$

has units $\dfrac{\text{mpg}}{\text{mph}}$

$= \dfrac{\text{mi/gal}}{\text{mi/hr}}$

$= \text{hr/gal}$

Interpret $e'(25) = 2.1$ hr/gal:

If we increase speed from 25 to 26, change in efficience is ≈ 2.1.

Restrict e to $[0,50]$ and let $f = e^{-1}$. What is $f'(22)$ and what does it mean?

$$f'(22) = \frac{1}{e'(\underbrace{25}_{f(22)})} = \frac{1}{2.1} \approx 0.476$$

$$\frac{\text{mpg}}{\text{mph}} = \frac{\text{mi/gal}}{\text{mi/hr}} = \text{hr/gal}$$

Note $f(\text{efficency})$ = speed b/w 0 and 50 with that assoc efficiency.