24. IX. 20

Review

- Limits:
  - from tables
  - from graphs
  - algebraically
  - laws
- Derivatives:
  - definition
  - tangent lines
  - rate of change
  - laws

    for exam, only power and linearity diff'n rules needed

Exam:
- 50 min in-class
- 2-sided page of notes
- 4 problems

E.g. We saw $\lim_{x \to 0} \frac{\sin x}{x} = 1$, so for x close to 0,

$$\frac{\sin x}{x} \approx 1 \iff \sin x \approx x.$$

For instance, $\sin 0.003 \approx 0.003$

$$\sin 0.003 = 0.002999995 \checkmark$$

Note $\sin x \approx x$ is a linear approximation of $\sin x$.

Is $y = x$ the tangent line to $y = \sin x$ at $(0, \sin 0) = (0, 0)$?

Since $y = x$ passes through $(0,0)$, "just" need to check if

$$m_{tan} = \frac{d}{dx} \sin x \Big|_{x=0} = 1.$$

Let $f(x) = \sin x$. Then

$$f'(0) = \lim_{h \to 0} \frac{\sin(0+h) - \sin(0)}{h}$$

$$= \lim_{h \to 0} \frac{\sin h}{h}$$

$$= 1.$$ ✓ That's what our limit was measuring all along!

Question For f differentiable, do we always have

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{f'(x)\, h} = 1 \; ?$$

Answer Only for $f'(x) \neq 0$!

Note More generally, $\displaystyle\lim_{x \to a} \frac{f(x)}{g(x)} = c \implies f(x) \approx c\,g(x)$ for x near a as long as $c \neq 0$.

linear approx'n to $y = f(x)$ near $x = a$ is

$$y - f(a) = f'(a)(x-a)$$

or $$y = f'(a) \cdot x - af'(a) + f(a)$$

Find tangent line to

$y = \sqrt{x} + x^2$ at $x = \cancel{2}\ 4$

A Let $f(x) = \sqrt{x} + x^2 = x^{1/2} + x^2$.

By linearity of diff'n and power rule,

$$f'(x) = \frac{1}{2}x^{-1/2} + 2x^1$$

$$= \frac{1}{2}x^{-1/2} + 2x.$$

Thus $f'(4) = \frac{1}{2} \cdot 4^{-1/2} + 2 \cdot 4 = \frac{1}{2\sqrt{4}} + 8 = \frac{1}{4} + 8$

$$= \frac{33}{4}.$$

The tangent line at $x=4$ is thus

$$y - f(4) = \frac{33}{4}(x-4)$$

$$y - \left(\sqrt{4} + 4^2\right) = \frac{33}{4}x - 33$$

$$y - 18 = \frac{33}{4}x - 33$$

$$y = \frac{33}{4}x - 15$$