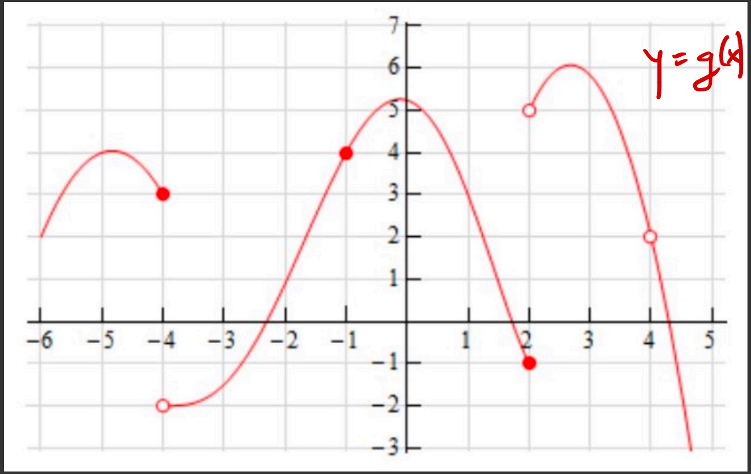
24. IX. 9

Quiz



Compute $\lim_{x \to a} g(x)$ for $a = -4, -1, 2, 4$.

a	-4	-1	2	4
$\lim_{x \to a} g(x)$	DNE	4	DNE	2
$\lim_{x \to a^+} g(x)$	-2	4	5	2
$\lim_{x \to a^-} g(x)$	3	4	-1	2

Some functions only have a limit from one "side".

$$\underbrace{\lim_{x \to a^+}}_{\text{x approaches a from the right}} f(x) \qquad \underbrace{\lim_{x \to a^-}}_{\text{x approaches a from the left}} f(x)$$

(See book for definitions.)

Problem What about left and right variants of limits above?

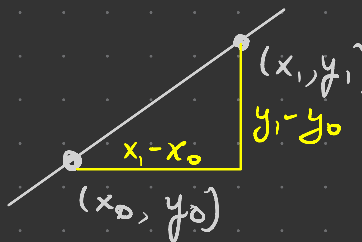
Problem (a) Graph $y = x^2$

(b) Use the following table to compute slopes of lines through $(1,1)$ and (x, x^2)

x	x^2	$\frac{x^2-1}{x-1}$
1.1	1.21	2.1
1.01	1.0201	2.01
1.001	1.002001	2.001
0.999	0.998001	1.999
0.99	0.9801	1.99
0.9	0.81	1.9

slope of line through (x_0, y_0) and (x_1, y_1) is

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{\text{rise}}{\text{run}}$$



(c) What is the limit of the secant slopes as $x \longrightarrow 1$?

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

or set $m(x) = \frac{x^2-1}{x-1}$ then $\lim_{x \to 1} m(x) = 2$

You just computed your first derivative!

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} =$$

$$\lim_{x \to 1} \frac{(x+1)(x-1)}{x-1}$$

$$= \lim_{x \to 1} (x+1)$$

$$= 1 + 1 = 2$$

slope $f'(a)$ = instantaneous rate of change of $f(x)$ as $x \to a$.

Discussion What is the relationship between

$$\lim_{x\to a} f(x), \quad \lim_{x\to a^+} f(x), \quad \text{and} \quad \lim_{x\to a^-} f(x) \ ?$$

In particular, what does $\lim_{x\to a} f(x) = L$ tell you about $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$? What does $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$ tell you about $\lim_{x\to a} f(x)$?

both equal L !

then $\lim_{x\to a} f(x)$ does not exist

Also, if $\lim_{x\to a^+} f(x) = L = \lim_{x\to a^-} f(x)$, then $\lim_{x\to a} f(x) = L$.

(In fact, equivalent!)

The sign function is $\text{sign}: \mathbb{R}_{\neq 0} \to \mathbb{R}$ given by

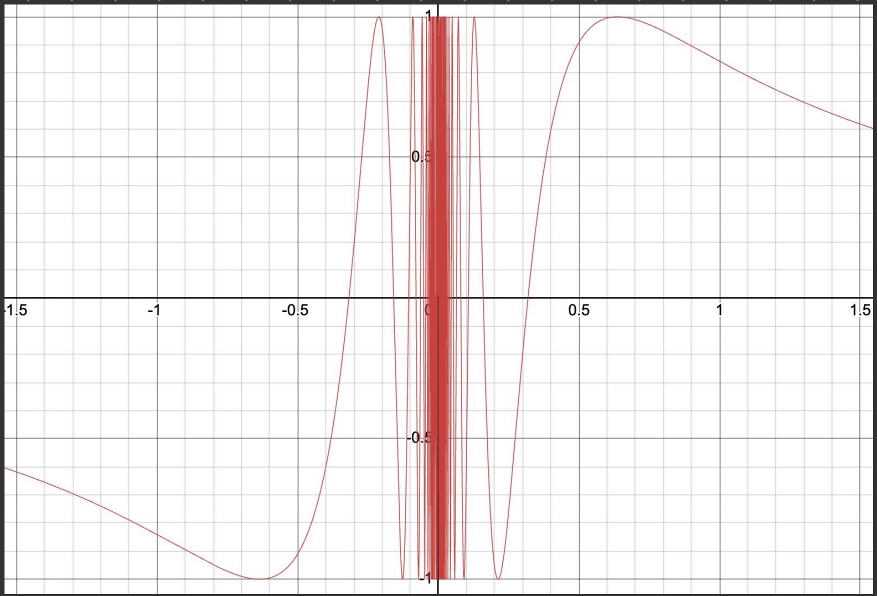
$$\text{sign}(x) = \frac{x}{|x|} \quad \text{for } x \neq 0.$$

(a) Graph $\text{sign}(x)$

(b) Compute $\lim\limits_{x \to 0^+} \text{sign}(x)$, $\lim\limits_{x \to 0^-} \text{sign}(x)$

(c) Fix a in $\mathbb{R}$. Where does $\text{sign}(x-a)$ fail to have a limit?

What about $\lim_{x \to 0^+} \sin\left(\frac{1}{x}\right)$?

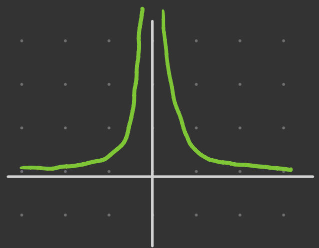


$y = \sin\left(\frac{1}{x}\right)$

$y = \sin(x)$

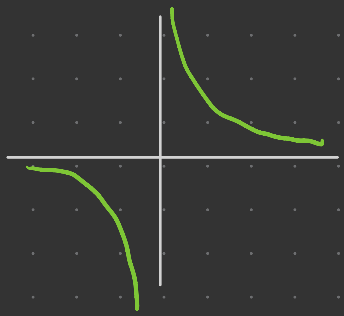
Infinite limits, limits at infinity

Consider $f(x) = \frac{1}{x^2}$:



Say $\lim_{x \to 0} f(x) = +\infty$ b/c as x gets close to 0 (from either side), $f(x)$ gets arbitrarily large.

What about $\lim_{x \to 0} \frac{1}{x}$?



DNE

Say $\lim_{x \to \infty} \frac{1}{x} = 0$ because as x gets arbitrarily large, $\frac{1}{x}$ gets closer and closer to 0.