24. $\overline{\underline{\text{IX}}}$. 6

Goals
- Define functions
- Define limits
- Explore limits

Defn A function f consists of
- a set of inputs (domain)
- a set of (potential) outputs (codomain) contains image := {values obtained by f}
- an assignment to each input x exactly one output $f(x)$

In Math 111, we will mostly think about functions with domain a subset of $\mathbb{R}$ (set of real #s), codomain $\mathbb{R}$.

E.g. • $f(x) = 3x^2$ is a function $\underbrace{\mathbb{R}}_{\text{domain}} \xrightarrow{f} \underbrace{\mathbb{R}}_{\text{codomain}}$

• $g(x) = \sqrt{x-1}$ is a function $[1,\infty) \longrightarrow \mathbb{R}$

$$[1,\infty) = \{x \in \mathbb{R} \mid 1 \le x\}$$

in

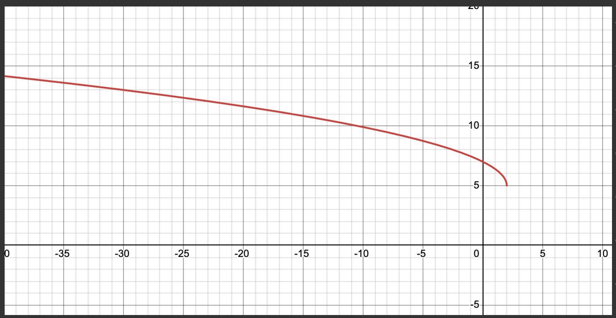
• $h(x) = \dfrac{x^2-1}{x-1}$ is a function $\mathbb{R}\setminus\{1\} \longrightarrow \mathbb{R}$.

$$= \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = x+1$$

$$\mathbb{R}\setminus\{1\} = \{x \text{ in } \mathbb{R} \mid x \neq 1\}$$

Question What are the domain and image of

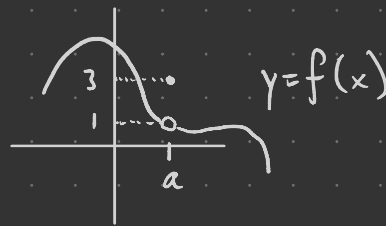
$$f(x) = \sqrt{4-2x} + 5 \;?$$



$$\text{image}(f) = \{f(x) \mid x \text{ in domain of } f\}$$

Limits answer the question "as x approaches (but does not equal!) a, what happens to $f(x)$?"

E.g.



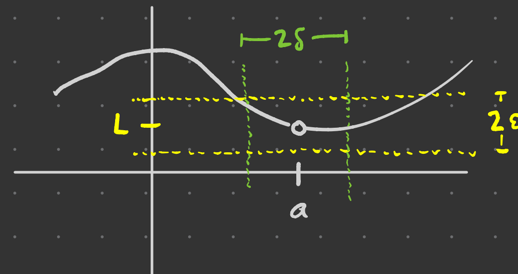
The limit of $f(x)$ as x approaches a is 1:

$$\lim_{x \to a} f(x) = 1.$$

Defn

- f a function defined on an open interval containing a, with the possible exception of a itself
- Say $\boxed{\lim_{x \to a} f(x) = L}$ when x gets closer to a but $\neq a$ implies $f(x)$ gets closer to L.

More formally, for any target $\varepsilon > 0$, we can find a bound $\delta > 0$ such that $f(x)$ is within ε of L for all x with δ of a (but $x \neq a$).



E.g. Consider $f(x) = \dfrac{\frac{1}{x} - 1}{x - 1}$. Determine $\lim_{x \to 1} f(x)$

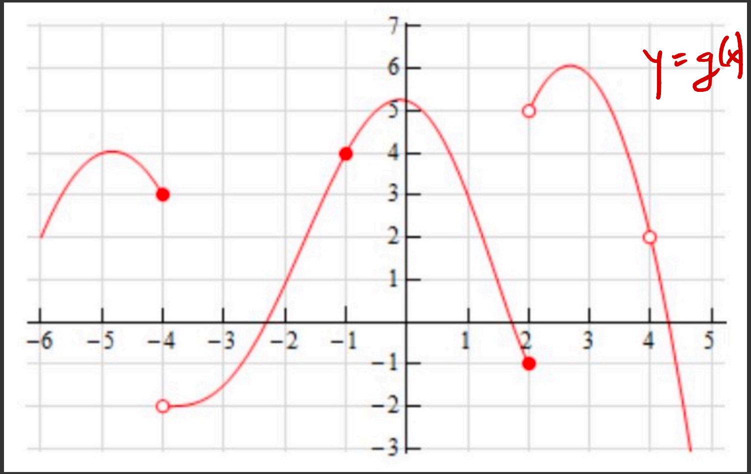
from this table:

x	$f(x) = \frac{\left(\frac{1}{x} - 1\right)}{x-1}$
0.9	-1.11111111111
0.99	-1.0101010101
0.999	-1.001001001
1.001	-0.999000999001
1.01	-0.990099009901
1.1	-0.909090909091

$= -1$

Problem Simplify $\dfrac{\frac{1}{x}-1}{x-1}$ algebraically to justify your answer.

Problem



Compute $\lim_{x \to a} g(x)$ for $a = -4, -1, 2, 4$.