MATH 111: CALCULUS
HOMEWORK DUE WEDNESDAY WEEK 14

Problem 1. Let $y = y(t)$ represent the size of a population varying over time t. Suppose that y satisfies

$$y' = rt(S - y)$$

for positive constants r, S. This model is similar to Newton's law of cooling, but now rt instead of just r is multiplying $(S - y)$.

(a) Find the most general solution to the above differential equation.
(b) What is the limit at $t \to \infty$ of $y(t)$?
(c) Find the particular solution when the initial population is $2S$.
(d) Find the particular solution when the initial population is $S/2$.

Problem 2. Suppose that a swimming pool contains 13,000 gallons of water and 350 pounds of salt. As it rains over the winter, water is drained from the pool at a rate of 225 gallons per week to keep the water level constant.

Let $s = s(t)$ denote the number of pounds of salt in the pool at time t (with t measured in weeks).

(a) In terms of s, what is the concentration of salt in the pool at time t (in pounds/gallon)?
(b) We have

$$s' = -(\text{number of pounds of salt lost per week}).$$

Write a differential equation for s' in terms of s.
(c) Solve the differential equation to find $s(t)$ with initial condition $s(0) = 350$ pounds.
(d) How many pounds of salt are lost after 28 weeks? Use a calculator to give an approximate solution.

If you owned this pool, this is the amount of salt you would have to add to the pool at the beginning of the swimming season!