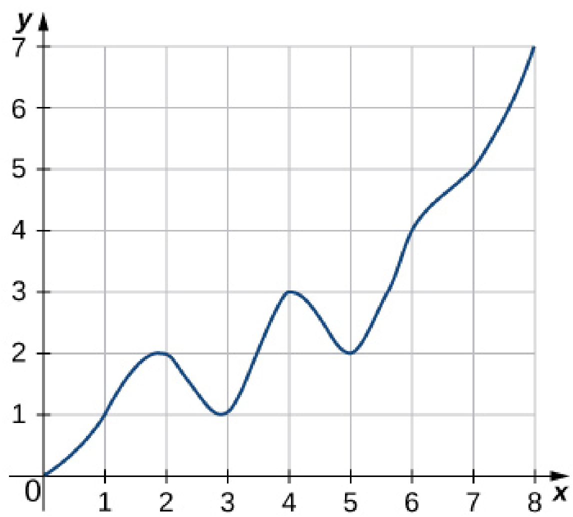
MATH 111: CALCULUS
HOMEWORK DUE WEDNESDAY WEEK 9

Problem 1. Let L_n denote the left-endpoint Riemann sum using n subintervals and let R_n denote the corresponding right-endpoint Riemann sum. Compute the indicated left and right Riemann sums for the given functions on the indicated interval.

(a) R_4 for $g(x) = \cos(\pi x)$ on $[0, 1]$
(b) L_8 for $x^2 - 2x + 1$ on $[0, 2]$

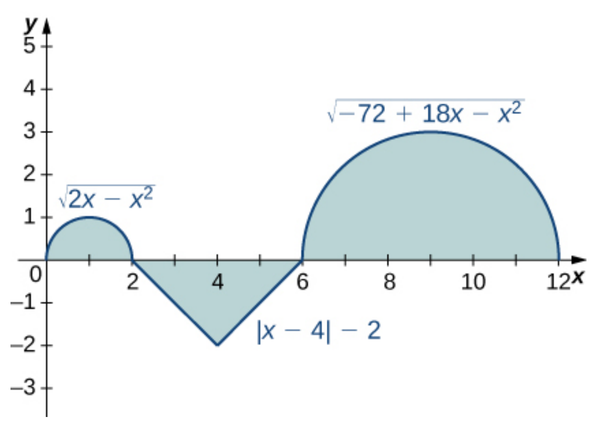
Problem 2. Estimate the area under the depicted curve by using the left Riemann sum L_8.



Problem 3. Suppose $f\colon [a, b] \to \mathbb{R}$ is an integrable function.

(a) Explain why $R_n - L_n = (f(b) - f(a)) \cdot \frac{b-a}{n}$.
(b) Suppose further that f is increasing on $[a, b]$. Explain why, in this case, the error betweeen either L_n or R_n and $\int_a^b f(x)\,dx$ is at most $(f(b) - f(a)) \cdot \frac{b-a}{n}$.

Problem 4. Use the information presented in the below graph of $y = h(x)$ to compute $\int_0^{12} h(x)\,dx$ geometrically.



Problem 5. Suppose that

$$A = \int_0^{2\pi} \sin^2 t\,dt \qquad \text{and} \qquad B = \int_0^{2\pi} \cos^2 t\,dt.$$

Show that $A + B = 2\pi$ and $A = B$.

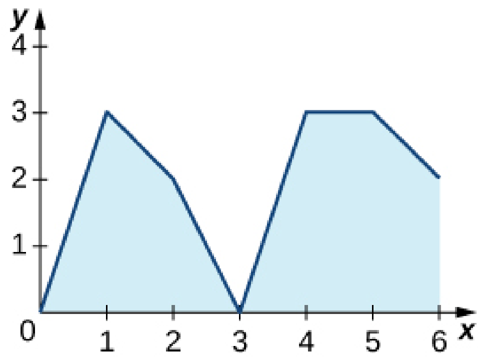
Problem 6. Suppose $f\colon [0,2] \to \mathbb{R}$ is given by $f(x) = \sqrt{4 - x^2}$.

(a) Use a geometric argument to compute the average value f_{ave} of f over $[0,2]$.
(b) Find a point c in $[0,2]$ such that $f(c) = f_{ave}$.

Problem 7. Use FTC1 to compute the following integrals:

(a) $\frac{d}{dx}\int_1^x e^{-t^2}\,dt$
(b) $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{t^2}{1+t^4}\,dt$

Problem 8. The graph of $y = \int_0^x f(t)\,dt$, where f is a piecewise constant function, is shown here.



(a) Over what intervals is f positive? negative? Over which intervals, if any, is f equal to 0?
(b) What are the maximum and minimum values of f?
(c) What is the average value of f?