MATH 111: CALCULUS
HOMEWORK DUE WEDNESDAY WEEK 4

Make sure to review the homework instructions in the syllabus before writing your solutions. In particular, show your work, write in complete sentences (but also aim for concise explanations), and explain your reasoning.

Problem 1. Find $\frac{dy}{dx}$ for the following functions:

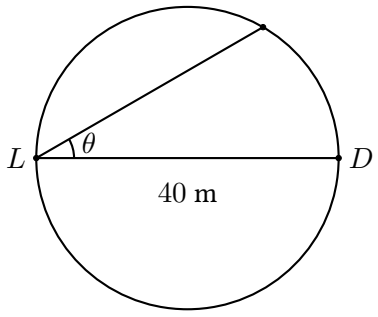
(a) $y = x - x^3 \sin x$,
(b) $y = \frac{e^{-x}}{x}$,
(c) $y = \sin x \tan x$,
(d) $y = 3^{\cos 3x}$,
(e) $y = \dfrac{\tan x}{1 - \sec x}$,
(f) $y = x^\pi \cdot \pi^x$,
(g) $y = \cos x(1 + \csc x)$.

Problem 2. Find all values x for which the graph of $f(x) = x - 2\cos x$ has tangent line of slope 2.

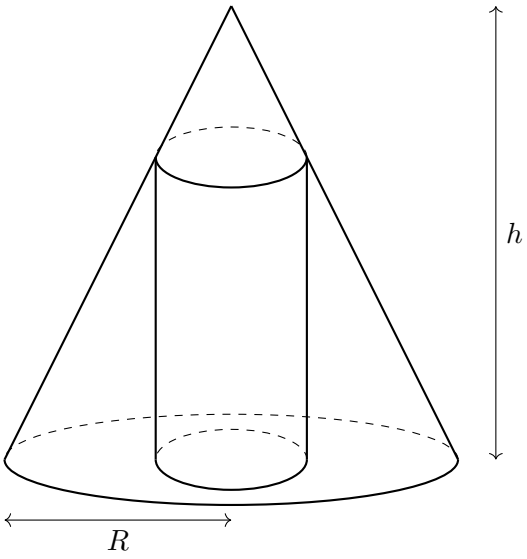
Problem 3. Find the local and absolute extrema for the following functions over the indicated domains:

(a) $a(x) = (x - x^2)^2$ over $[-1, 1]$,
(b) $b(\theta) = 4\sin\theta - 3\cos\theta$ over $[0, 2\pi]$,
(c) $c(x) = x^3(1 - x)^6$ over $(-\infty, \infty)$.

Problem 4. Consider a lifeguard L at a circular pool with diameter 40m. They must reach someone who is drowning at the exact opposite side of the pool (position D in the diagram). The lifeguard swims with a speed v and runs around the pool with a speed $3v$. If the lifeguard first swims at angle θ across the pool (as indicated in the diagram) and then runs to D around the edge of the pool, at what angle θ should they swim in order to minimize the time of their journey.



Problem 5. Find the largest volume of a cylinder that fits into a cone that has base radius R and height h as indicated in the diagram on the next page.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