

MATH 111: CALCULUS
HOMEWORK DUE WEDNESDAY WEEK 3

Make sure to review the homework instructions in the syllabus before writing your solutions. In particular, show your work, write in complete sentences (but also aim for concise explanations), and explain your reasoning.

Problem 1. Use limit laws and algebra to evaluate the following the following limits.

- (a) $\lim_{x \rightarrow 1} \frac{x^3 + 3x^2 + 5}{4 - 7x}$
 (b) $\lim_{x \rightarrow -2} (4x^2 - 1)$
 (c) $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 2x}$

Problem 2. (a) Fix a nonzero constant a . Use limit laws and algebra to evaluate the limit

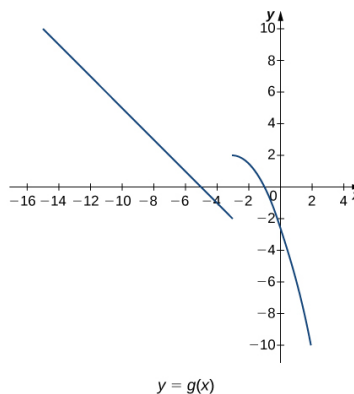
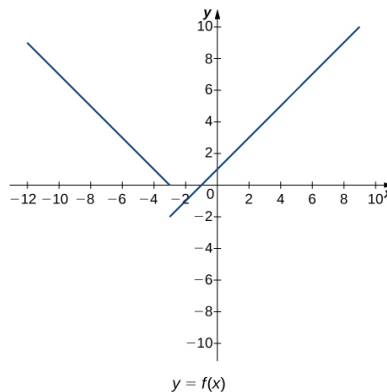
$$\lim_{h \rightarrow 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}.$$

(Your answer should be a formula involving a .)

(b) For what function $f(x)$ does your work in part (a) determine $f'(a)$.

Problem 3. Use the graphs of $y = f(x)$ and $y = g(x)$ below and the limit laws to evaluate

$$\lim_{x \rightarrow 0} \frac{f(x)g(x)}{3}.$$



Problem 4. (a) Use the fact that $(x - 2)^2 \geq 0$ for all real x to explain why $2x - 1 \leq x^2 - 2x + 3$ for all real x .

(b) Determine the limits of both $2x - 1$ and $x^2 - 2x + 3$ as $x \rightarrow 2$.

(c) Suppose that $g(x)$ is a function defined on an open interval containing 2, and suppose that $2x - 1 \leq g(x) \leq x^2 - 2x + 3$. Evaluate $\lim_{x \rightarrow 2} g(x)$.

Problem 5. (a) Let $f(x) = 1 - x - x^2$. Use the definition of the derivative and limit laws to determine $f'(0)$.

(b) Use your answer from part (a) to determine the equation of the tangent line to $y = f(x)$ at $x = 0$.

(c) Use desmos or a similar tool to plot $y = f(x)$ and your tangent line to check your solution.